QUADRATIC EQUATION

A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator the smaller the fraction.........
Tolstoy, Count Lev Nikolgevich

1. Polynomial :

A function f defined by $f(x) = a_nx^n + a_{n-1}x^{n-1} + ...+ a_1x + a_0$
where $a_0, a_1, a_2, ..., a_n \in R$ is called a polynomial of degree n with real coefficients ($a_n \neq 0, n \in W$). If $a_0, a_1, a_2, ..., a_n \in C$, it is called a polynomial with complex coefficients.

2. Quadratic polynomial & Quadratic equation :

A polynomial of degree 2 is known as quadratic polynomial. Any equation f(x) = 0, where f is a quadratic polynomial, is called a quadratic equation. The general form of a quadratic equation is

$ax^2 + bx + c = 0$ ....(i)

Where a, b, c are real numbers, $a \neq 0$.
If a = 0, then equation (i) becomes linear equation.

3. Difference between equation & identity :

If a statement is true for all the values of the variable, such statements are called as identities. If the statement is true for some or no values of the variable, such statements are called as equations.

Example :
(i) $(x + 3)^2 = x^2 + 6x + 9$ is an identity
(ii) $(x + 3)^2 = x^2 + 6x + 8$, is an equation having no root.
(iii) $(x + 3)^2 = x^2 + 5x + 8$, is an equation having – 1 as its root.

A quadratic equation has exactly two roots which may be real (equal or unequal) or imaginary.
$ax^2 + bx + c = 0$ is:

★	a quadratic equation if	$a \neq 0$	Two Roots
★	a linear equation if	$a = 0, b \neq 0$	One Root
★	a contradiction if	$a = b = 0, c \neq 0$	No Root
★	an identity if	$a = b = c = 0$	Infinite Roots

If $ax^2 + bx + c = 0$ is satisfied by three distinct values of 'x', then it is an identity.

Example # 1 : (i) $3x^2 + 2x – 1 = 0$ is a quadratic equation here a = 3.
(ii) $(x + 1)^2 = x^2 + 2x + 1$ is an identity in x.

Solution : Here highest power of x in the given relation is 2 and this relation is satisfied by three different values x = 0, x = 1 and x = – 1 and hence it is an identity because a polynomial equation of n^{th} degree cannot have more than n distinct roots.

4. Relation Between Roots & Co-efficients:

(i) The solutions of quadratic equation, $ax^2 + bx + c = 0$, ($a \neq 0$) is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression, $b^2 – 4ac \equiv D$ is called discriminant of quadratic equation.

(ii) If α, β are the roots of quadratic equation,
$ax^2 + bx + c = 0$ ....(i)
then equation (i) can be written as
$a(x – \alpha)(x – \beta) = 0$ or $ax^2 – a(\alpha + \beta)x + a\alpha\beta = 0$ ......(ii)
equations (i) and (ii) are identical,

$\therefore$ by comparing the coefficients sum of the roots, $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

and product of the roots, $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

(iii) Dividing the equation (i) by a, $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\Rightarrow$ $x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$ $\Rightarrow$ $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow$ x^2 – (sum of the roots) x + (product of the roots) = 0

Hence we conclude that the quadratic equation whose roots are α & β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Example # 2 : If α and β are the roots of $ax^2 + bx + c = 0$, find the equation whose roots are $\alpha+2$ and $\beta+2$.

Solution : Replacing x by x – 2 in the given equation, the required equation is
$a(x-2)^2 + b(x-2) + c = 0$ i.e., $ax^2 - (4a - b)x + (4a - 2b + c) = 0$.

Example # 3 : The coefficient of x in the quadratic equation $x^2 + px + q = 0$ was taken as 17 in place of 13, its roots were found to be – 2 and – 15. Find the roots of the original equation.

Solution : Here q = (– 2) × (– 15) = 30, correct value of p = 13. Hence original equation is
$x^2 + 13x + 30 = 0$ as $(x + 10)(x + 3) = 0$
$\therefore$ roots are – 10, – 3

Self practice problems :

(1) If α, β are the roots of the quadratic equation $cx^2 - 2bx + 4a = 0$ then find the quadratic equation whose roots are

(i) $\frac{\alpha}{2}, \frac{\beta}{2}$ (ii) α^2, β^2 (iii) $\alpha + 1, \beta + 1$

(iv) $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}$ (v) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

(2) If r be the ratio of the roots of the equation $ax^2 + bx + c = 0$, show that $\frac{(r+1)^2}{r} = \frac{b^2}{ac}$.

Answers : (1)
(i) $cx^2 - bx + a = 0$
(ii) $c^2x^2 + 4(b^2 - 2ac)x + 16a^2 = 0$
(iii) $cx^2 - 2x(b + c) + (4a + 2b + c) = 0$
(iv) $(c - 2b + 4a)x^2 + 2(4a - c)x + (c + 2b + 4a) = 0$
(v) $4acx^2 + 4(b^2 - 2ac)x + 4ac = 0$

5. Theory Of Equations :

If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the roots of the equation;

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ where $a_0, a_1, ..., a_n$ are all real & $a_0 \neq 0$ then,

$\sum \alpha_1 = -\frac{a_1}{a_0}, \sum \alpha_1\alpha_2 = +\frac{a_2}{a_0}, \sum \alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}, ..., \alpha_1\alpha_2\alpha_3.......\alpha_n = (-1)^n\frac{a_n}{a_0}$

Note : (i) If α is a root of the equation f(x) = 0, then the polynomial f(x) is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of f(x) and conversely.

(ii) Every equation of n^{th} degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If the coefficients of the equation f(x) = 0 are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. imaginary roots occur in conjugate pairs.

(iv) An equation of odd degree will have odd number of real roots and an equation of even degree will have even numbers of real roots.

(v) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not square of a rational number.

(vi) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then f(x) = 0 must have odd number of real roots (also atleast one real root) between 'a' and 'b'.

(vii) Every equation f(x) = 0 of degree odd has atleast one real root of a sign opposite to that of its last term. (If coefficient of highest degree term is positive).

Example # 4 : If $2x^3 + 3x^2 + 5x + 6 = 0$ has roots α, β, γ then find $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$.

Solution : Using relation between roots and coefficients, we get

$$\alpha + \beta + \gamma = = -\frac{3}{2}, \qquad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{5}{2}, \qquad \alpha\beta\gamma = -\frac{6}{2} = -3.$$

Self practice problems :

(3) If $2p^3 - 9pq + 27r = 0$ then prove that the roots of the equations $rx^3 - qx^2 + px - 1 = 0$ are in H.P.

(4) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$ then find the equation whose roots are
(a) $2\alpha + 2\beta + \gamma,\ \alpha + 2\beta + 2\gamma,\ 2\alpha + \beta + 2\gamma$
(b) $-\frac{r}{\alpha}, -\frac{r}{\beta}, -\frac{r}{\gamma}$
(c) $(\alpha + \beta)^2, (\beta + \gamma)^2, (\gamma + \alpha)^2$
(d) $-\alpha^3, -\beta^3, -\gamma^3$

Answers : (4) (a) $x^3 + qx - r = 0$ (b) $x^3 - qx^2 - r^2 = 0$
(c) $x^3 + 2qx^2 + q^2 x - r^2 = 0$ (d) $x^3 - 3x^2r + (3r^2 + q^3)\,x - r^3 = 0$

6. Nature of Roots:

Consider the quadratic equation, $ax^2 + bx + c = 0$ having α, β as its roots;

$D \equiv b^2 - 4ac$

$D = 0$
Roots are equal i.e. $\alpha = \beta = -b/2a$
& the quadratic expression can be expressed
as a perfect square of a linear polynomial

$D \neq 0$
Roots are unequal

$a, b, c \in R$ & $D > 0$
Roots are real

$a, b, c \in R$ & $D < 0$
Roots are imaginary $\alpha = p + iq$, $\beta = p - iq$

$a, b, c \in Q$ &
D is square of a rational number
$\Rightarrow$ Roots are rational
$\downarrow$
$a = 1$, $b, c \in I$ & D is square of an integer
$\Rightarrow$ Roots are integral.

$a, b, c \in Q$ &
D is not square of a rational number
$\Rightarrow$ Roots are irrational
i.e. $\alpha = p + \sqrt{q}$, $\beta = p - \sqrt{q}$

Example # 5 : For what values of m the equation $(1 + m)\,x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has equal roots.

Solution : Given equation is $(1 + m)\,x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ ....(i)
Let D be the discriminant of equation (i).
Roots of equation (i) will be equal if D = 0.
or $4(1 + 3m)^2 - 4(1 + m)(1 + 8m) = 0$
or $4(1 + 9m^2 + 6m - 1 - 9m - 8m^2) = 0$
or $m^2 - 3m = 0$ or, $m(m - 3) = 0$
$\therefore$ $m = 0, 3$.

Example # 6 : Find all the integral values of a for which the quadratic equation (x – a) (x – 10) + 1 = 0 has integral roots.

Solution : Here the equation is $x^2 - (a + 10)x + 10a + 1 = 0$. Since integral roots will always be rational it means D should be a perfect square.

From (i) $D = a^2 - 20a + 96$.

$\Rightarrow \quad D = (a - 10)^2 - 4 \qquad \Rightarrow \quad 4 = (a - 10)^2 - D$

If D is a perfect square it means we want difference of two perfect square as 4 which is possible

only when $(a - 10)^2 = 4$ and D = 0.

$\Rightarrow \quad (a - 10) = \pm 2 \qquad \Rightarrow \quad a = 12, 8$

Example # 7 : If the roots of the equation (x – a) (x – b) – k = 0 be c and d, then prove that the roots of the equation (x – c) (x – d) + k = 0, are a and b.

Solution : By given condition $(x - a)(x - b) - k \equiv (x - c)(x - d)$

or $(x - c)(x - d) + k \equiv (x - a)(x - b)$

Above shows that the roots of (x – c) (x – d) + k = 0 are a and b.

Example # 8 : Determine 'a' such that $x^2 - 11x + a$ and $x^2 - 14x + 2a$ may have a common factor.

Solution : Let $x - \alpha$ be a common factor of $x^2 - 11x + a$ and $x^2 - 14x + 2a$.

Then $x = \alpha$ will satisfy the equations $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$.

$\therefore \quad \alpha^2 - 11\alpha + a = 0$ and $\quad \alpha^2 - 14\alpha + 2a = 0$

Solving (i) and (ii) by cross multiplication method, we get a = 0, 24.

Example # 9 : Show that the expression $x^2 + 2(a + b + c)x + 3(bc + ca + ab)$ will be a perfect square if a = b = c.

Solution : Given quadratic expression will be a perfect square if the discriminant of its corresponding equation is zero.

i.e. $\quad 4(a + b + c)^2 - 4.3\,(bc + ca + ab) = 0$

or $\quad (a + b + c)^2 - 3(bc + ca + ab) = 0$

or $\quad \frac{1}{2}\left((a - b)^2 + (b - c)^2 + (c - a)^2\right) = 0$

which is possible only when a = b = c.

Self practice problems :

(5) For what values of 'k' the expression $(4 - k)x^2 + 2(k + 2)x + 8k + 1$ will be a perfect square ?

(6) If $(x - \alpha)$ be a factor common to $a_1x^2 + b_1x + c$ and $a_2x^2 + b_2x + c$, then prove that $\alpha(a_1 - a_2) = b_2 - b_1$.

(7) If $3x^2 + 2\alpha xy + 2y^2 + 2ax - 4y + 1$ can be resolved into two linear factors, Prove that 'α' is a root of the equation $x^2 + 4ax + 2a^2 + 6 = 0$.

(8) Let $4x^2 - 4(\alpha - 2)x + \alpha - 2 = 0$ $(\alpha \in R)$ be a quadratic equation. Find the values of 'α' for which
(i) Both roots are real and distinct.
(ii) Both roots are equal.
(iii) Both roots are imaginary
(iv) Both roots are opposite in sign.
(v) Both roots are equal in magnitude but opposite in sign.

(9) If $P(x) = ax^2 + bx + c$, and $Q(x) = -ax^2 + dx + c$, $ac \neq 0$ then prove that P(x) . Q(x) = 0 has atleast two real roots.

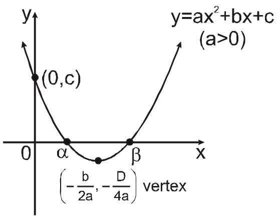
Answers. (5) 0, 3
(8) (i) $(-\infty, 2) \cup (3, \infty)$ (ii) $\alpha \in \{2, 3\}$ (iii) (2, 3) (iv) $(-\infty, 2)$ (v) ϕ

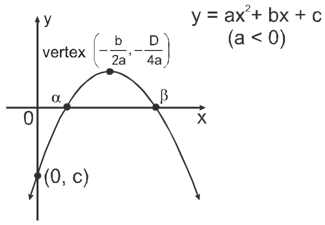
7. Graph of Quadratic Expression :

★ the graph between x, y is always a parabola.

★ the co–ordinate of vertex are $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

★ If a > 0 then the shape of the parabola is concave upwards & if a < 0 then the shape of the parabola is concave downwards.





★ the parabola intersect the y–axis at point (0, c).

★ the x–co–ordinate of point of intersection of parabola with x–axis are the real roots of the quadratic equation f (x) = 0. Hence the parabola may or may not intersect the x–axis.

8. Range of Quadratic Expression $f(x) = ax^2 + bx + c$.

(i) Range :

If a > 0 $\Rightarrow$ $f(x) \in \left[-\frac{D}{4a}, \infty\right)$

If a < 0 $\Rightarrow$ $f(x) \in \left(-\infty, -\frac{D}{4a}\right]$

Hence maximum and minimum values of the expression f (x) is $-\frac{D}{4a}$ in respective cases and it occurs at $x = -\frac{b}{2a}$ (at vertex).

(ii) Range in restricted domain:

Given $x \in [x_1, x_2]$

(a) If $-\frac{b}{2a} \notin [x_1, x_2]$ then,

$f(x) \in [\min\{f(x_1), f(x_2)\}, \max\{f(x_1), f(x_2)\}]$

(b) If $-\frac{b}{2a} \in [x_1, x_2]$ then,

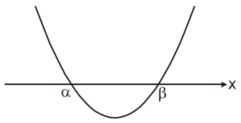
$$f(x) \in \left[\min\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}, \max\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}\right]$$

9. Sign of Quadratic Expressions :

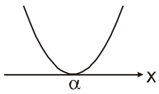
The value of expression $f(x) = ax^2 + bx + c$ at $x = x_0$ is equal to y–co–ordinate of the point on parabola $y = ax^2 + bx + c$ whose x–co–ordinate is x_0. Hence if the point lies above the x–axis for some $x = x_0$, then $f(x_0) > 0$ and vice–versa.

We get six different positions of the graph with respect to x–axis as shown.

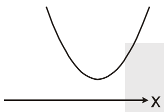
Conclusions :

(i) 

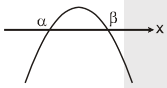
(a) $a > 0$
(b) $D > 0$
(c) Roots are real & distinct.
(d) $f(x) > 0$ in $x \in (-\infty, \alpha) \cup (\beta, \infty)$
(e) $f(x) < 0$ in $x \in (\alpha, \beta)$

(ii) 

(a) $a > 0$
(b) $D = 0$
(c) Roots are real & equal.
(d) $f(x) > 0$ in $x \in R - \{\alpha\}$

(iii) 

(a) $a > 0$
(b) $D < 0$
(c) Roots are imaginary.
(d) $f(x) > 0 \ \forall \ x \in R$.

(iv) 

(a) $a < 0$
(b) $D > 0$
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(d) $f(x) < 0$ in $x \in (-\infty, \alpha) \cup (\beta, \infty)$
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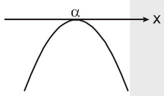
(v) 

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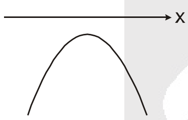
(vi) 

(a) $a < 0$
(b) $D < 0$
(c) Roots are imaginary.
(d) $f(x) < 0 \ \forall \ x \in R$.

Example # 10 : If $c < 0$ and $ax^2 + bx + c = 0$ does not have any real roots then prove that
(i) $a - b + c < 0$ (ii) $9a + 3b + c < 0$.

Solution : $c < 0$ and $D < 0$ $\Rightarrow$ $f(x) = ax^2 + bx + c < 0$ for all $x \in R$
$\Rightarrow$ $f(-1) = a - b + c < 0$
and $f(3) = 9a + 3b + c < 0$

Example # 11 : Find the range of $f(x) = x^2 - 5x + 6$.

Solution : minimum of $f(x) = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$ $= -\left(\frac{25-24}{4}\right)$ at $x = \frac{5}{2}$ $= -\frac{1}{4}$

maximum of $f(x) \to \infty$

Hence range is $\left[-\frac{1}{4}, \infty\right)$

Example # 12 : Find the range of rational expression y = $\frac{x^2 - x + 4}{x^2 + x + 4}$ if x is real.

Solution : $y = \frac{x^2 - x + 4}{x^2 + x + 4}$ $\Rightarrow$ $(y - 1)x^2 + (y + 1)x + 4(y - 1) = 0$ ....(i)

case-I : if $y \neq 1$, then equation (i) is quadratic in x

and $\because$ x is real

$\therefore$ $D \geq 0$ $\Rightarrow$ $(y + 1)^2 - 16(y - 1)^2 \geq 0$ $\Rightarrow$ $(5y - 3)(3y - 5) \leq 0$

$\therefore$ $y \in \left[\frac{3}{5}, \frac{5}{3}\right] - \{1\}$

case-II : if y = 1, then equation becomes

2x = 0 $\Rightarrow$ x = 0 which is possible as x is real.

$\therefore$ Range $\left[\frac{3}{5}, \frac{5}{3}\right]$

Example # 13 : Find the range of y = $\frac{x+3}{2x^2 + 3x + 9}$, if x is real.

Solution : $y = \frac{x+3}{2x^2 + 3x + 9}$

$\Rightarrow$ $2yx^2 + (3y - 1)x + 3(3y - 1) = 0$ ....(i)

case-I : if $y \neq 0$, then equation (i) is quadratic in x

$\because$ x is real

$\therefore$ $D \geq 0$

$\Rightarrow$ $(3y - 1)^2 - 24y(3y - 1) \geq 0$

$\Rightarrow$ $(3y - 1)(21y + 1) \leq 0$

$y \in \left[-\frac{1}{21}, \frac{1}{3}\right] - \{0\}$

case-II : if y = 0, then equation becomes

x = –3 which is possible as x is real

$\therefore$ Range $y \in \left[-\frac{1}{21}, \frac{1}{3}\right]$

Self practice problems :

(10) If c > 0 and $ax^2 + 2bx + 3c = 0$ does not have any real roots then prove that

(i) $4a - 4b + 3c > 0$ (ii) $a + 6b + 27c > 0$ (iii) $a + 2b + 6c > 0$

(11) If f(x) = (x – a) (x – b), then show that $f(x) \geq -\frac{(a-b)^2}{4}$.

(12) Find the least integral value of 'k' for which the quadratic polynomial $(k - 1)x^2 + 8x + k + 5 > 0 \ \forall\ x \in R$.

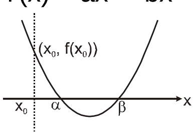
(13) Find the range of the expression $\frac{x^2 + 34x - 71}{x^2 + 2x - 7}$, if x is a real.

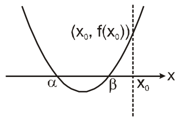
(14) Find the interval in which 'm' lies so that the expression $\frac{mx^2 + 3x - 4}{-4x^2 + 3x + m}$ can take all real values, $x \in R$.

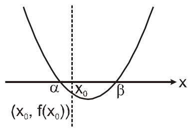
Answers : (12) k = 4 (13) $(-\infty, 5] \cup [9, \infty)$ (14) $m \in (1, 7)$

10. Location of Roots :

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

(i) 

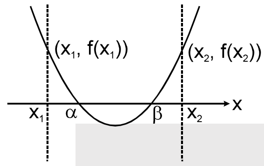
(ii) 

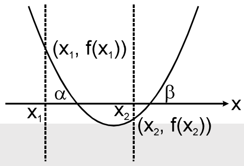
(iii) 

(i) Conditions for both the roots of f (x) = 0 to be greater than a specified number 'x_0' are $b^2 - 4ac \geq 0$ & $f(x_0) > 0$ & $(-b/2a) > x_0$.

(ii) Conditions for both the roots of f (x) = 0 to be smaller than a specified number 'x_0' are $b^2 - 4ac \geq 0$ & $f(x_0) > 0$ & $(-b/2a) < x_0$.

(iii) Conditions for a number 'x_0' to lie between the roots of f (x) = 0 is $f(x_0) < 0$.

(iv) 

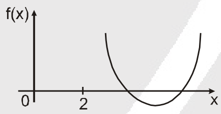
(v) 

(iv) Conditions that both roots of f (x) = 0 to be confined between the numbers x_1 and x_2, $(x_1 < x_2)$ are $b^2 - 4ac \geq 0$ & $f(x_1) > 0$ & $f(x_2) > 0$ & $x_1 < (-b/2a) < x_2$.

(v) Conditions for exactly one root of f (x) = 0 to lie in the interval (x_1, x_2) i.e. $x_1 < x < x_2$ is $f(x_1) . f(x_2) < 0$.

Example # 14 : Let $x^2 - (m - 3)x + m = 0$ $(m \in R)$ be a quadratic equation, then find the values of 'm' for which

(a) both the roots are greater than 2.
(b) both roots are positive.
(c) one root is positive and other is negative.
(d) One root is greater than 2 and other smaller than 1
(e) Roots are equal in magnitude and opposite in sign.
(f) both roots lie in the interval (1, 2)

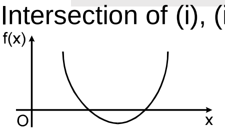
Solution : (a) 

Condition - I : $D \geq 0$ $\Rightarrow$ $(m - 3)^2 - 4m \geq 0$ $\Rightarrow$ $m^2 - 10m + 9 \geq 0$
$\Rightarrow$ $(m - 1)(m - 9) \geq 0$
$\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$ ......(i)

Condition - II : $f(2) > 0$ $\Rightarrow$ $4 - (m - 3)2 + m > 0 \Rightarrow$ $m < 10$ ......(ii)

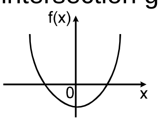
Condition - III : $-\dfrac{b}{2a} > 2$ $\Rightarrow$ $\dfrac{m-3}{2} > 2 \Rightarrow$ $m > 7$ ......(iii)

Intersection of (i), (ii) and (iii) gives $m \in [9, 10)$

(b) 

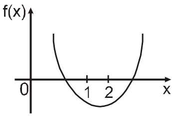
Condition - I $D \geq 0$ $\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$
Condition - II $f(0) > 0$ $\Rightarrow$ $m > 0$
Condition - III $-\dfrac{b}{2a} > 0$ $\Rightarrow$ $\dfrac{m-3}{2} > 0$ $\Rightarrow$ $m > 3$

intersection gives $m \in [9, \infty)$ Ans.

(c) 

Condition - I $f(0) < 0$ $\Rightarrow$ $m < 0$ Ans.

(d)



Condition - I $f(1) < 0$ $\Rightarrow$ $4 < 0$ $\Rightarrow$ $m \in \phi$

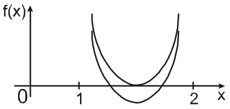
Condition - II $f(2) < 0$ $\Rightarrow$ $m > 10$

Intersection gives $m \in \phi$ Ans.

(e) sum of roots = 0 $\Rightarrow$ $m = 3$

and $f(0) < 0$ $\Rightarrow$ $m < 0$ $\therefore$ $m \in \phi$ **Ans.**

(f)



Condition - I $D \geq 0$ $\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$

Condition - II $f(1) > 0$ $\Rightarrow$ $1 - (m - 3) + m > 0$ $\Rightarrow$ $4 > 0$ which is true $\forall$ $m \in R$

Condition - III $f(2) > 0$ $\Rightarrow$ $m < 10$

Condition - IV $1 < -\frac{b}{2a} < 2$ $\Rightarrow$ $1 < \frac{m-3}{2} < 2$ $\Rightarrow$ $5 < m < 7$

intersection gives $m \in \phi$ **Ans.**

Example # 15 : Find all the values of 'a' for which both the roots of the equation $(a - 2)x^2 - 2ax + a = 0$ lies in the interval $(-2, 1)$.

Solution : Case-I : $f(-2) > 0$ $\Rightarrow$ $4(a - 2) + 4a + a > 0$

$9a - 8 > 0$ $\Rightarrow$ $a > \frac{8}{9}$

$f(1) > 0$ $\Rightarrow$ $a - 2 - 2a + a > 0$

$-2 > 0$ not possible $\therefore$ $a \in \phi$

Case-II : $a - 2 < 0$ $\Rightarrow$ $a < 2$

$f(-2) < 0$ $\Rightarrow$ $a < \frac{8}{9}$

$f(1) < 0$ $\Rightarrow$ $a \in R$

$-2 < \frac{b}{2a} - < 1$ $\Rightarrow$ $a < \frac{4}{3}$

$D \geq 0$ $\Rightarrow$ $a \geq 0$

intersection gives $a \in \left[0, \frac{8}{9}\right)$

complete solution $a \in \left[0, \frac{8}{9}\right) \cup \{2\}$

Self practice problems :

(15) Let $x^2 - 2(a - 1)x + a - 1 = 0$ $(a \in R)$ be a quadratic equation, then find the value of 'a' for which

(a) Both the roots are positive
(b) Both the roots are negative
(c) Both the roots are opposite in sign.
(d) Both the roots are greater than 1.
(e) Both the roots are smaller than 1.
(f) One root is small than 1 and the other root is greater than 1.

(16) Find the values of p for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2.

(17) Find the values of 'α' for which 6 lies between the roots of the equation $x^2 + 2(\alpha - 3)x + 9 = 0$.

(18) Let $x^2 - 2(a - 1)x + a - 1 = 0$ $(a \in R)$ be a quadratic equation, then find the values of 'a' for which

(i) Exactly one root lies in (0, 1).
(ii) Both roots lies in (0, 1).
(iii) Atleast one root lies in (0, 1).
(iv) One root is greater than 1 and other root is smaller than 0.

(19) Find the values of a, for which the quadratic expression $ax^2 + (a - 2)x - 2$ is negative for exactly two integral values of x.

Answers : (15) (a) $[2, \infty)$ (b) ϕ (c) $(-\infty, 1)$ (d) ϕ (e) $(-\infty, 1]$ (f) $(2, \infty)$

(16) $(-\infty, -1)$ (17) $\left(-\infty, -\frac{3}{4}\right)$

(18) (i) $(-\infty, 1) \cup (2, \infty)$ (ii) ϕ (iii) $(-\infty, 1) \cup (2, \infty)$ (iv) ϕ

(19) $[1, 2)$

11. Common Roots:

Consider two quadratic equations, $a_1x^2 + b_1x + c_1 = 0$ & $a_2x^2 + b_2x + c_2 = 0$.

(i) If two quadratic equations have both roots common, then the equations are identical and their co-efficient are in proportion.

i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(ii) If only one root is common, then the common root 'α' will be :

$$\alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} = \frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}$$

Hence the condition for one common root is :

$$\Rightarrow \quad (c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$$

Note : If f(x) = 0 & g(x) = 0 are two polynomial equation having some common root(s) then those common root(s) is/are also the root(s) of $h(x) \equiv a f(x) + bg(x) = 0$.

Example # 16 : If $x^2 - ax + b = 0$ and $x^2 - px + q = 0$ have a root in common and the second equation has equal roots, show that $b + q = \frac{ap}{2}$.

Solution : Given equations are : $x^2 - ax + b = 0$ .... (i)

and $x^2 - px + q = 0$. .... (ii)

Let α be the common root. Then roots of equation (ii) will be α and α. Let β be the other root of equation (i). Thus roots of equation (i) are α, β and those of equation (ii) are α, α.

Now $\alpha + \beta = a$ .... (iii)

$\alpha\beta = b$ .... (iv)

$2\alpha = p$ .... (v)

$\alpha^2 = q$ .... (vi)

L.H.S. $= b + q = \alpha\beta + \alpha^2 = \alpha(\alpha + \beta)$ .... (vii)

and R.H.S. $= \frac{ap}{2} = \frac{(\alpha+\beta)\,2\alpha}{2} = \alpha(\alpha + \beta)$ .... (viii)

from (vii) and (viii), L.H.S. = R.H.S.

Example # 17 : If a, b, c $\in$ R and equations $ax^2 + bx + c = 0$ and $x^2 + 2x + 9 = 0$ have a common root, show that a : b : c = 1 : 2 : 9.

Solution : Given equations are : $x^2 + 2x + 9 = 0$ ....(i)

and $ax^2 + bx + c = 0$ ....(ii)

Clearly roots of equation (i) are imaginary since equation (i) and (ii) have a common root, therefore common root must be imaginary and hence both roots will be common.

Therefore equations (i) and (ii) are identical

$\therefore \quad \frac{a}{1} = \frac{b}{2} = \frac{c}{9}$

$\therefore \quad$ a : b : c = 1 : 2 : 9

Self practice problems :

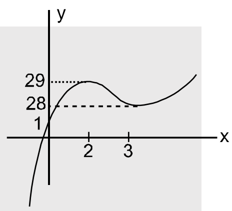
(20) If the equations $ax^2 + bx + c = 0$ and $x^3 + x - 2 = 0$ have two common roots then show that $2a = 2b = c$.

(21) If $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in A.P. show that a_1, b_1, c_1 are in G.P.

12. Graphs of Polynomials

$y = a_nx^n + ... + a_1x + a_0$. The points where $y' = 0$ are called turning points which are critical in plotting the graph.

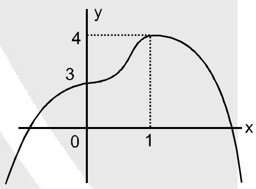
Example # 18 : Draw the graph of $y = 2x^3 - 15x^2 + 36x + 1$



Solution. $y' = 6x^2 - 30x + 36 = 6(x - 3)(x - 2)$

x	2	3	∞	–∞
y	29	28	∞	–∞

Example # 19 : Draw the graph of $y = -3x^4 + 4x^3 + 3$,



Solution. $y' = -12x^3 + 12x$

$y' = -12x^2 (x - 1)$

x	0	1	∞	–∞
y	3	4	–∞	–∞

Exercise-1

Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Relation between the roots and coefficients ; Quadratic Equation

A-1. For what value of 'a', the equation $(a^2 - a - 2)x^2 + (a^2 - 4)x + (a^2 - 3a + 2) = 0$, will have more than two solutions ? Does there exist a real value of 'x' for which the above equation will be an identity in 'a' ?

A-2. If α and β are the roots of the equation $2x^2 + 3x + 4 = 0$, then find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

A-3. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then find the equation whose roots are given by

(i) $\alpha + \frac{1}{\beta}, \beta + \frac{1}{\alpha}$ (ii) $\alpha^2 + 2, \beta^2 + 2$

A-4. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then find the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

A-5. In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as – 10 in place of – 11 and the roots were found to be 4 and 6. Find the roots of the correct equation.

A-6. **(i)** Find the value of the expression $2x^3 + 2x^2 - 7x + 72$ when $x = \frac{3+5\sqrt{-1}}{2}$.

(ii) Find the value of the expression $2x^3 + 2x^2 - 7x + 72$ when $x = \frac{-1+\sqrt{15}}{2}$

(iii) Solve the following equation $2^{2x} + 2^{x+2} - 32 = 0$

A-7. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β

A-8. If α, β are roots of $x^2 - px + q = 0$ and $\alpha - 2$, $\beta + 2$ are roots of $x^2 - px + r = 0$, then prove that $16q + (r + 4 - q)^2 = 4p^2$.

A-9. If one root of the equation $ax^2 + bx + c = 0$ is equal to n^{th} power of the other root, then show that $(ac^n)^{1/(n+1)} + (a^nc)^{1/(n+1)} + b = 0$.

A-10. If the sum of the roots of quadratic equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is -1, then find the product of the roots.

A-11. Find the least prime integral value of '2a' such that the roots α, β of the equation $2x^2 + 6x + a = 0$ satisfy the inequality $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$.

Section (B) : Relation between roots and coefficients ; Higher Degree Equations

B-1. If α and β be two real roots of the equation $x^3 + px^2 + qx + r = 0$ $(r \neq 0)$ satisfying the relation $\alpha\beta + 1 = 0$, then prove that $r^2 + pr + q + 1 = 0$.

B-2. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right)$.

B-3. **(i)** Solve the equation $24x^3 - 14x^2 - 63x + \lambda = 0$, one root being double of another. Hence find the value(s) of λ.

(ii) Solve the equation $18x^3 + 81x^2 + \lambda x + 60 = 0$, one root being half the sum of the other two. Hence find the value of λ.

B-4. If α, β, γ are roots of equation $x^3 - 6x^2 + 10x - 3 = 0$, then find cubic equation with roots $2\alpha + 1$, $2\beta + 1$, $2\gamma + 1$.

B-5. If α, β and γ are roots of $2x^3 + x^2 - 7 = 0$, then find the value of $\sum_{\alpha,\beta,\gamma}\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

B-6. Find the roots of $4x^3 + 20x^2 - 23x + 6 = 0$ if two of its roots are equal.

Section (C) : Nature of Roots

C-1. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$ (where p, q $\in$ R and $i^2 = -1$), then find the ordered pair (p, q).

C-2. If the roots of the equation $x^2 - 2cx + ab = 0$ are real and unequal, then prove that the roots of $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ will be imaginary.

C-3. For what values of k the expression $kx^2 + (k + 1)x + 2$ will be a perfect square of a linear polynomial.

C-4. Show that if roots of equation $(a^2 - bc)x^2 + 2(b^2 - ac)x + c^2 - ab = 0$ are equal, then either $b = 0$ or $a^3 + b^3 + c^3 = 3abc$

C-5. If a, b, c $\in$ R, then prove that the roots of the equation $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$ are always real and cannot have roots if a = b = c.

C-6. If the roots of the equation $\frac{1}{(x+p)} + \frac{1}{(x+q)} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then show that $p + q = 2r$ and that the product of the roots is equal to $(-1/2)(p^2 + q^2)$.

C-7. **(i)** If $-2 + i\beta$ is a root of $x^3 + 63x + \lambda = 0$ (where $\beta \in R - \{0\}$, $\lambda \in R$ and $i^2 = -1$), then find roots of equation.

(ii) If $\frac{-1}{2} + i\beta$, is a root of $2x^3 + bx^2 + 3x + 1 = 0$ (where $b, \beta \in R - \{0\}$ and $i^2 = -1$), then find the value(s) of b.

C-8. Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$, one root being $-1 + \sqrt{-1}$.

C-9. Draw graph of $y = 12x^3 - 4x^2 - 3x + 1$. Hence find number of positive zeroes.

Section (D) : Range of quadratic expression and sign of quadratic expression

D-1. Draw the graph of the following expressions :

(i) $y = x^2 + 4x + 3$ (ii) $y = 9x^2 + 6x + 1$ (iii) $y = -2x^2 + x - 1$

D-2. Find the range of following quadratic expressions :
(i) $f(x) = -x^2 + 2x + 3 \quad \forall x \in R$
(ii) $f(x) = x^2 - 2x + 3 \quad \forall x \in [0, 3]$
(iii) $f(x) = x^2 - 4x + 6 \quad \forall x \in (0, 1]$

D-3. If x be real, then find the range of the following rational expressions :
(i) $y = \frac{x^2 + x + 1}{x^2 + 1}$ **(ii)** $y = \frac{x^2 - 2x + 9}{x^2 - 2x - 9}$

D-4. Find the range of values of k, such that f(x) = $\frac{kx^2 + 2(k+1)x + (9k+4)}{x^2 - 8x + 17}$ is always negative.

D-5. $x^2 + (a - b) x + (1 - a - b) = 0$, $a, b \in R$. Find the condition on ' a ' for which
(i) Both roots of the equation are real and unequal $\forall b \in R$.
(ii) Roots are imaginary $\forall b \in R$

Section (E) : Location of Roots

E-1. If both roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3, then show that $a > 11/9$.

E-2. Find all the values of 'K' for which one root of the equation $x^2 - (K + 1) x + K^2 + K - 8 = 0$, exceeds 2 & the other root is smaller than 2.

E-3. Find all the real values of 'a', so that the roots of the equation $(a^2 - a + 2) x^2 + 2(a - 3) x + 9 (a^4 - 16) = 0$ are of opposite sign.

E-4. Find all the values of 'a', so that exactly one root of the equation $x^2 - 2ax + a^2 - 1 = 0$, lies between the numbers 2 and 4, and no root of the equation is either equal to 2 or equal to 4.

E-5. If α & β are the two distinct roots of $x^2 + 2 (K - 3) x + 9 = 0$, then find the values of K such that $\alpha, \beta \in (-6, 1)$.

Section (F) : Common Roots & Graphs of Polynomials

F-1. If one of the roots of the equation $ax^2 + b x + c = 0$ be reciprocal of one of the roots of $a_1 x^2 + b_1 x + c_1 = 0$, then prove that $(a a_1 - c c_1)^2 = (b c_1 - a b_1) (b_1 c - a_1 b)$.

F-2. Find the value of 'a' so that $x^2 - 11 x + a = 0$ and $x^2 - 14x + 2a = 0$ have a common root.

F-3. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and a, b, c are non-zero real numbers, then find the value of $\frac{a^3 + b^3 + c^3}{abc}$.

F-4. If $x^2 + px + q = 0$ and $x^2 + qx + p = 0$, $(p \neq q)$ have a common root, show that $1 + p + q = 0$; show that their other roots are the roots of the equation $x^2 + x + pq = 0$.

F-5. Draw the graphs of following :
(i) $y = 2x^3 + 9x^2 - 24x + 15$ (ii) $y = -3x^4 + 4x^3 + 12x^2 - 2$

F-6. Find values of 'k' if equation $x^3 - 3x^2 + 2 = k$ has
(i) 3 real roots (ii) 1 real root

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Relation between the roots and coefficients quadratic equation

A-1. The roots of the equation $(b – c) x^2 + (c – a) x + (a – b) = 0$ are

(A) $\frac{c-a}{b-c}$, 1 (B) $\frac{a-b}{b-c}$, 1 (C) $\frac{b-c}{a-b}$, 1 (D) $\frac{c-a}{a-b}$, 1

A-2. If α, β are the roots of quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma) . (\alpha - \delta)$ is equal to :

(A) q + r (B) q − r (C) − (q + r) (D) − (p + q + r)

A-3. Two real numbers α & β are such that $\alpha + \beta = 3$, $\alpha - \beta = 4$, then α & β are the roots of the quadratic equation:

(A) $4x^2 – 12x – 7 = 0$ (B) $4x^2 – 12x + 7 = 0$ (C) $4x^2 – 12x + 25 = 0$ (D) none of these

A-4. For the equation $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to:

(A) 1/3 (B) 1 (C) 3 (D) 2/3

A-5. Consider the following statements :

S_1 : If the roots of $x^2 – bx + c = 0$ are two consecutive integers, then value of $b^2 – 4c$ is equal to 1.

S_2 : If α, β are roots of $x^2 – x + 3 = 0$ then value of $\alpha^4 + \beta^4$ is equal 7.

S_3 : If α, β, γ are the roots of $x^3 – 7x^2 + 16x – 12 = 0$ then value of $\alpha^2 + \beta^2 + \gamma^2$ is equal to 17.

State, in order, whether S_1, S_2, S_3 are true or false

(A) TTT (B) FTF (C) TFT (D) FTT

Section (B) : Relation between roots and coefficients ; Higher Degree Equations

B-1. If two roots of the equation $x^3 - px^2 + qx - r = 0$, $(r \neq 0)$ are equal in magnitude but opposite in sign, then:

(A) pr = q (B) qr = p (C) pq = r (D) None of these

B-2. If α, β & γ are the roots of the equation $x^3 – x – 1 = 0$ then, $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ has the value equal to:

(A) zero (B) –1 (C) –7 (D) 1

B-3. Let α, β, γ be the roots of $(x – a)(x – b)(x – c) = d$, $d \neq 0$, then the roots of the equation $(x – \alpha)(x – \beta)(x – \gamma) + d = 0$ are :

(A) a + 1, b + 1, c + 1 (B) a, b, c (C) a – 1, b – 1, c – 1 (D) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

B-4. If α, β, γ are the roots of the equation $x^3 + ax + b = 0$ then value of $\frac{\alpha^3+\beta^3+\gamma^3}{\alpha^2+\beta^2+\gamma^2}$ is equal to :

(A) $\frac{3b}{2a}$ (B) $\frac{-3b}{2a}$ (C) 3b (D) 2b

B-5. If two of the roots of equation $x^4 – 2x^3 + ax^2 + 8x + b = 0$ are equal in magnitude but opposite in sign, then value of 4a + b is equal to :

(A) 16 (B) 8 (C) –16 (D) –8

Section (C) : Nature of Roots

C-1. If one roots of equation $x^2 - \sqrt{3}\ x + \lambda = 0$, $\lambda \in R$ is $\sqrt{3} + 2$ then other root is
(A) $\sqrt{3} - 2$ (B) -2 (C) $2 - \sqrt{3}$ (D) 2

C-2. If roots of equation $2x^2 + bx + c = 0$; $b, c \in R$, are real & distinct then the roots of equation $2cx^2 + (b - 4c)\ x + 2c - b + 1 = 0$ are
(A) imaginary (B) equal (C) real and distinct (D) can't say

C-3. Let one root of the equation $x^2 + \ell x + m = 0$ is square of other root. If $m \in R$ then
(A) $\ell \in \left(-\infty, \frac{1}{4}\right] \cup \{1\}$ (B) $\ell \in (-\infty, 0]$ (C) $\ell \in \left(-\infty, \frac{1}{9}\right]$ (D) $\ell \in \left(\frac{1}{4}, 1\right]$

C-4. If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the quadratic equation $ax^2 + bx + c = 0$ are
(A) Irrational (B) Rational & different (C) Complex conjugate (D) Rational & equal

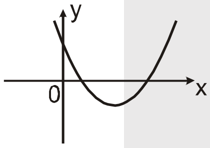
C-5. Let a and b be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has
(A) real roots (B) imaginary roots (C) exactly one root (D) none of these

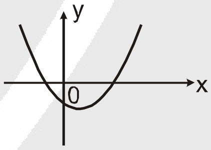
C-6. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. Total number of different values of 'n' so that the given equation has integral roots, is
(A) 4 (B) 6 (C) 8 (D) 3

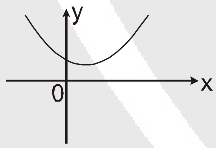
Section (D) : Range of quadratic expression and sign of quadratic expression

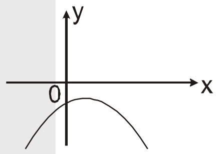
D-1. If α & β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta^2 < \alpha^2$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < \alpha^2 < \beta^2$

D-2. Which of the following graph represents the expression $f(x) = a\ x^2 + b\ x + c$ ($a \neq 0$) when $a > 0$, $b < 0$ & $c < 0$?

(A) 

(B) 

(C) 

(D) 

D-3. The expression $y = ax^2 + bx + c$ has always the same sign as of 'a' if :
(A) $4ac < b^2$ (B) $4ac > b^2$ (C) $4ac = b^2$ (D) $ac < b^2$

D-4. The entire graph of the expression $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if
(A) $k < 7$ (B) $-5 < k < 7$ (C) $k > -5$ (D) none of these

D-5. If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is:
(A) positive (B) negative (C) zero (D) depends on the sign of b

D-6. If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is
(A) $\frac{3}{2}$ (B) $\frac{9}{4}$ (C) $-\frac{9}{4}$ (D) 1

D-7. If $y = -2x^2 - 6x + 9$, then
(A) maximum value of y is –11 and it occurs at x = 2
(B) minimum value of y is –11 and it occurs at x = 2
(C) maximum value of y is 13.5 and it occurs at x = –1.5
(D) minimum value of y is 13.5 and it occurs at x = –1.5

D-8. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min f(x) > max g(x), then the relation between b and c, is
(A) no relation (B) $0 < c < b/2$ (C) $c^2 < 2b$ (D) $c^2 > 2b^2$

Section (E) : Location of Roots

E-1. If b > a, then the equation (x – a) (x – b) – 1 = 0, has:
(A) both roots in [a, b]
(B) both roots in (– ∞, a)
(C) both roots in [b, ∞)
(D) one root in (– ∞, a) & other in (b, ∞)

E-2. If α, β are the roots of the quadratic equation $x^2 - 2p(x - 4) - 15 = 0$, then the set of values of 'p' for which one root is less than 1 & the other root is greater than 2 is:
(A) (7/3, ∞)
(B) (–∞, 7/3)
(C) x ∈ R
(D) none of these

E-3. If α, β be the roots of $4x^2 - 16x + \lambda = 0$, where λ ∈ R, such that 1 < α < 2 and 2 < β < 3, then the number of integral solutions of λ is
(A) 5
(B) 6
(C) 2
(D) 3

E-4. Set of real values of k if the equation $x^2 - (k-1)x + k^2 = 0$ has atleast one root in (1,2) is
(A) (2, 4)
(B) [–1, 1/3]
(C) {3}
(D) ϕ

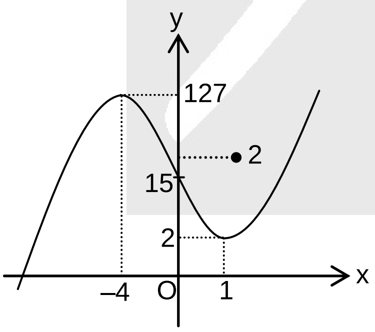
Section (F) : Common Roots & Graphs of Polynomials

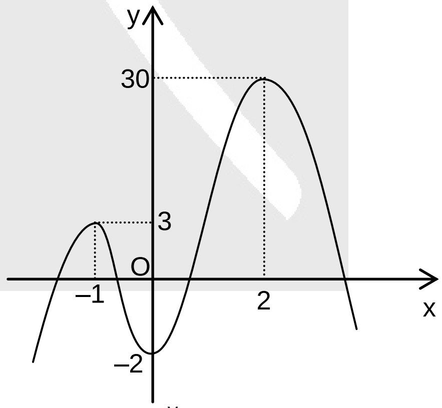
F-1. If the equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ have both roots common, then the value of (2r – p) is
(A) 0
(B) 1/2
(C) 1
(D) none of these

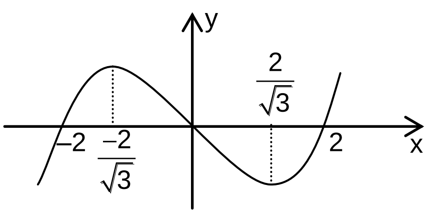
F-2. If $3x^2 - 17x + 10 = 0$ and $x^2 - 5x + \lambda = 0$ has a common root, then sum of all possible real values of λ is
(A) 0
(B) $-\frac{29}{9}$
(C) $\frac{26}{9}$
(D) $\frac{29}{3}$

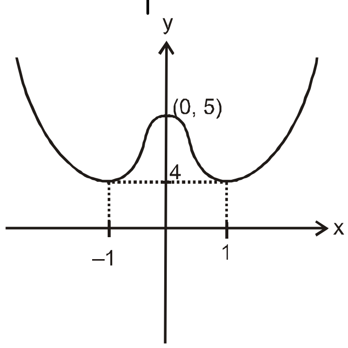
F-3. If a, b, p, q are non–zero real numbers, then two equations $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have :
(A) no common root
(B) one common root if $2a^2 + b^2 = p^2 + q^2$
(C) two common roots if 3 pq = 2 ab
(D) two common roots if 3 qb = 2 ap

F-4. The graphs of $y = \frac{x^3 - 4x}{4}$ is

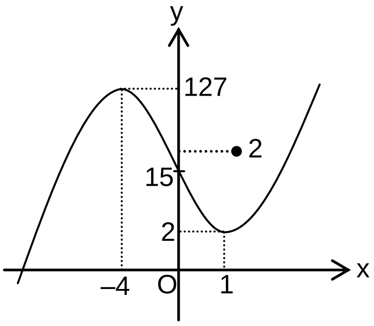
(A)


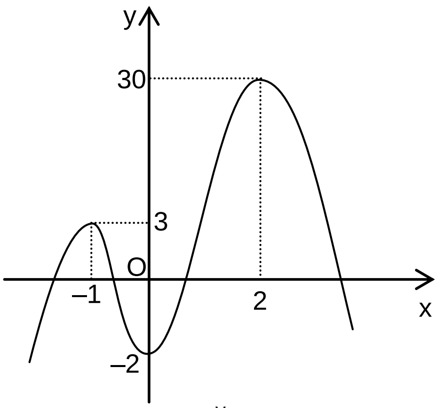
(B)


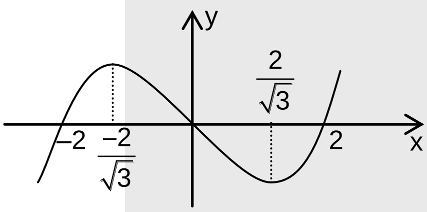
(C)


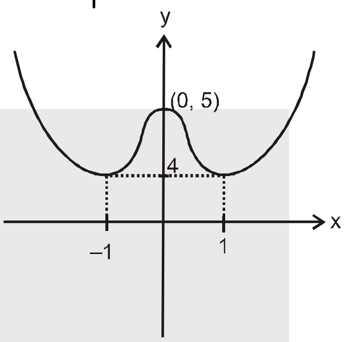
(D)


F-5. The graphs of $y = x^4 - 2x^2 + 5$ is

(A)


(B)


(C)


(D)


PART - III : MATCH THE COLUMN

1. **Column – I** **Column – II**

(A) If α, $\alpha + 4$ are two roots of $x^2 - 8x + k = 0$, then possible value of k is — (p) 4

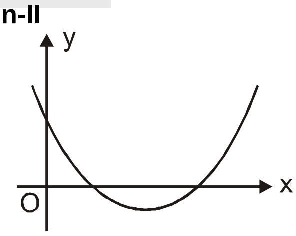
(B) If α, β are roots of $x^2 + 2x - 4 = 0$ and $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $x^2 + qx + r = 0$ then value of $\frac{-3}{q+r}$ is — (q) 0

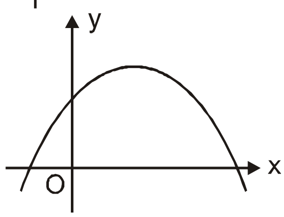
(C) If α, β are roots of $ax^2 + c = 0$, $ac \neq 0$, then $\alpha^3 + \beta^3$ is equal to — (r) 12

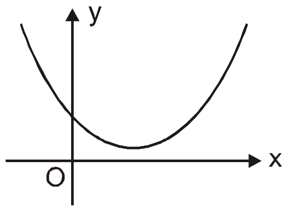
(D) If roots of $x^2 - kx + 36 = 0$ are Integers then number of values of k = — (s) 10

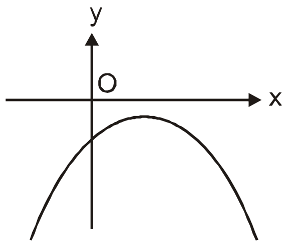
2. If graph of the expression $f(x) = ax^2 + bx + c$ $(a \neq 0)$ are given in column-II, then Match the items in column-I with in column-II (where $D = b^2 - 4ac$)

Column-I **Column-II**

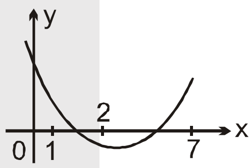
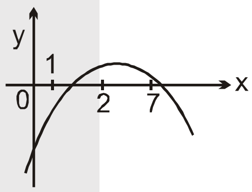
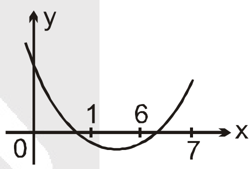
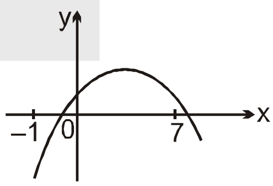
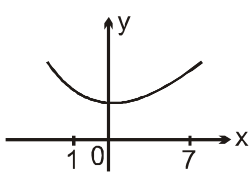
(A) $\frac{abc}{D} > 0$ — (p)


(B) $\frac{abc}{D} < 0$ — (q)


(C) abc > 0 (r) 

(D) abc < 0 (s) 

3. Let $y = Q(x) = ax^2 + bx + c$ be a quadratic expression. Match the inequalities in **Column-I** with possible graphs in **Column-II.**

Column-I	**Column-II**
(A) $Q(x) > 0, \forall x \in (2, 7)$	(p) 
(B) $Q(x) > 0, \forall x \in (-\infty, 1)$	(q) 
(C) $Q(x) < 0, \forall x \in (1, 6)$	(r) 
(D) $Q(x) < 0, \forall x \in (-\infty, -1)$	(s) 
	(t) 

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Let $a > 0$, $b > 0$ & $c > 0$. Then both the roots of the equation $ax^2 + bx + c = 0$
(A) are real & negative
(B) have negative real parts
(C) are rational numbers
(D) have positive real parts

2. If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by atmost 2m, then b lies in the interval
(A) $(a^2 - m^2, a^2)$
(B) $[a^2 - m^2, a^2)$
(C) $(a^2, a^2 + m^2)$
(D) none of these

3. The set of possible values of λ for which $x^2 - (\lambda^2 - 5\lambda + 5)x + (2\lambda^2 - 3\lambda - 4) = 0$ has roots, whose sum and product are both less than 1, is
(A) $\left(-1\ ,\ \frac{5}{2}\right)$
(B) (1, 4)
(C) $\left[1\ ,\ \frac{5}{2}\right]$
(D) $\left(1\ ,\ \frac{5}{2}\right)$

4. If p, q, r, s $\in$ R, then equaton $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx - 2q) = 0$ has
(A) 6 real roots
(B) atleast two real roots
(C) 2 real and 4 imaginary roots
(D) 4 real and 2 imaginary roots

5. If coefficients of biquadratic equation are all distinct and belong to the set {–9, – 5, 3, 4, 7}, then equation has
(A) atleast two real roots
(B) four real roots, two are conjugate surds and other two are also conjugate surds
(C) four imaginary roots
(D) None of these

6. Let p, q, r, s $\in$ R, $x^2 + px + q = 0$, $x^2 + rx + s = 0$ such that $2(q + s) = pr$ then
(A) atleast one of the equation have real roots.
(B) either both equations have imaginary roots or both equations have real roots.
(C) one of equations have real roots and other equation have imaginary roots
(D) atleast one of the equations have imaginary roots.

7. The equation, $\pi^x = -2x^2 + 6x - 9$ has:
(A) no solution
(B) one solution
(C) two solutions
(D) infinite solutions

8. If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1$ for all $x \in R$, then λ belongs to the interval
(A) (–2, 1)
(B) $\left[-2, \frac{2}{5}\right)$
(C) $\left(\frac{2}{5}, 1\right)$
(D) none of these

9. Let conditions C_1 and C_2 be defined as follows : $C_1 : b^2 - 4ac \ge 0$, C_2 : a, –b, c are of same sign. The roots of $ax^2 + bx + c = 0$ are real and positive, if
(A) both C_1 and C_2 are satisfied
(B) only C_2 is satisfied
(C) only C_1 is satisfied
(D) none of these

10. If 'x' is real, then $\frac{x^2 - x + c}{x^2 + x + 2c}$ can take all real values if :
(A) $c \in [0, 6]$
(B) $c \in [-6, 0]$
(C) $c \in (-\infty, -6) \cup (0, \infty)$
(D) $c \in (-6, 0)$

11. If both roots of the quadratic equation $(2 - x)(x + 1) = p$ are distinct & positive, then complet set of values of p is:
(A) $(2, \infty)$ (B) $(2, 9/4)$ (C) $(-\infty, -2)$ (D) $(-\infty, \infty)$

12. If two roots of the equation $(a - 1)(x^2 + x + 1)^2 - (a + 1)(x^4 + x^2 + 1) = 0$ are real and distinct, then 'a' lies in the interval
(A) $(-2, 2)$ (B) $(-\infty, -2) \cup (2, \infty)$ (C) $(2, \infty)$ (D) $(-\infty, -2)$

13. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then the ordered pair (x_1, x_2) is:
(A) $(-5, -7)$ (B) $(1, -1)$ (C) $(-1, 1)$ (D) $(5, 7)$

14. If a, b, c are real and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval:
(A) $\left[\frac{1}{2}, 2\right]$ (B) $[0, 2]$ (C) $\left[-\frac{1}{2}, 1\right]$ (D) $\left[-1, \frac{1}{2}\right]$

PART - II : NUMERICAL VALUE QUESTIONS

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. Find sum of square of real roots of equation $x(x + 1)(x + 2)(x + 3) = 120$

2. Find product of all real values of x satisfying $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$

3. If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Then find the value of $(a - c)(b - c)(a + d)(b + d)/(q^2 - p^2)$.

4. α, β are roots of the equation $\lambda(x^2 - x) + x + 5 = 0$. If λ_1 and λ_2 are the two values of λ for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4$, then the value of $\left(\frac{\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}}{15}\right)$ is

5. Let one root of equation $(\ell - m)x^2 + \ell x + 1 = 0$ be double of the other. If ℓ be real and $m \le k$ then find the least value of k.

6. Let α, β be the roots of the equation $x^2 + ax + b = 0$ and γ, δ be the roots of $x^2 - ax + b - 2 = 0$. If $\alpha\beta\gamma\delta = 24$ and $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{11}{5}$, then find the value of a.

7. If $a > b > 0$ and $a^3 + b^3 + 27ab = 729$ then the quadratic equation $ax^2 + bx - 9 = 0$ has roots α, β $(\alpha < \beta)$. Find the value of $4\beta - a\alpha$.

8. Let α and β be roots of $x^2 - 6(5t^2 - 3t + 7)x - 2 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \ge 1$, then find the minimum value of $\frac{a_{100} - 2a_{98}}{a_{99}}$ (where $t \in R$)

9. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - Kx^3 + Kx^2 + Lx + M = 0$, where K, L & M are real numbers, then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is – n. Find the value of n.

10. Consider $y = \frac{2x}{1+x^2}$, where x is real, then the range of expression $y^2 + y - 2$ is [a, b]. Find the value of (b –a).

11. If the roots of the equation $3x^3 + Px^2 + Qx - 37 = 0$ are each one more than the roots of the equaton $x^3 - Ax^2 + Bx - C = 0$, where A, B, C, P & Q are constants, then the value of A + B + C is equal to :

12. If one root of the equation $t^2 - (12x)t - (f(x) + 64x) = 0$ is twice of other, then find the maximum value of the function f(x), where $x \in R$.

13. The values of k, for which the equation $x^2 + 2(k - 1)x + k + 5 = 0$ possess atleast one positive root, are $(-\infty, -b]$. Find value of b.

14. Find the least value of 'a' for which atleast one of the roots of the equation $x^2 - (a - 3)x + a = 0$ is greater than 2.

15. If the quadratic equations $3x^2 + ax + 1 = 0$ & $2x^2 + bx + 1 = 0$ have a common root, then the value of the expression $5ab - 2a^2 - 3b^2$ is

16. The equations $3x^2 - 7ax + b = 0$, $x^3 - px^2 + qx = 0$, where a, b, p, q $\in R - \{0\}$ have one common root & the second equation has two equal roots. Find value of $\frac{3q+b}{aq}$.

17. If x – y and y – 2x are two factors of the expression $x^3 - 3x^2y + \lambda xy^2 + \mu y^3$, then $\lambda^2 + \mu^2$ is

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Possible values of 'p' for which the equation $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ does not possess more than two roots is/are

(A) 0 (B) 1 (C) 2 (D) 4

2. If a, b are non-zero real numbers and α, β the roots of $x^2 + ax + b = 0$, then

(A) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$

(B) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$

(C) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$

(D) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

3. If α, β are the roots of $ax^2 + bx + c = 0$ $(a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of, $Ax^2 + Bx + C = 0$ $(A \neq 0)$ for some constant δ, then

(A) $\delta = \frac{1}{2}\left(\frac{B}{A} - \frac{b}{a}\right)$ (B) $\delta = \frac{1}{2}\left(\frac{b}{a} - \frac{B}{A}\right)$

(C) $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$ (D) $\frac{b^2 + 4ac}{a^2} = \frac{B^2 + 4AC}{A^2}$

4. If one root of the equation $4x^2 + 2x - 1 = 0$ is 'α', then

(A) α can be equal to $\frac{-1+\sqrt{5}}{4}$ (B) α can be equal to $\frac{1+\sqrt{5}}{4}$

(C) other root is $4\alpha^3 - 3\alpha$. (D) other root is $4\alpha^3 + 3\alpha$

5. If α, β are roots of $x^2 + 3x + 1 = 0$, then

(A) $(7 - \alpha)(7 - \beta) = 0$ (B) $(2 - \alpha)(2 - \beta) = 11$

(C) $\dfrac{\alpha^2}{3\alpha+1} + \dfrac{\beta^2}{3\beta+1} = -2$ (D) $\left(\dfrac{\alpha}{1+\beta}\right)^2 + \left(\dfrac{\beta}{\alpha+1}\right)^2 = 18$

6. If both roots of $x^2 - 32x + c = 0$ are prime numbers then possible values of c are

(A) 60 (B) 87 (C) 247 (D) 231

7. Let $f(x) = x^2 - a(x + 1) - b = 0$, $a, b \in R - \{0\}$, $a + b \neq 0$. If α and β are roots of equation $f(x) = 0$, then the value of $\dfrac{1}{\alpha^2 - a\alpha} + \dfrac{1}{\beta^2 - a\beta} - \dfrac{2}{a+b}$ is equal to

(A) 0 (B) $f(a) + a + b$ (C) $f(b) + a + b$ (D) $f\left(\dfrac{a}{2}\right) + \dfrac{a^2}{4} + a + b$

8. If f(x) is a polynomial of degree three with leading coefficient 1 such that $f(1) = 1$, $f(2) = 4$, $f(3) = 9$, then

(A) $f(4) = 22$ (B) $f\left(\dfrac{6}{5}\right) = \left(\dfrac{6}{5}\right)^3$

(C) $f(x) = x^3$ holds for exactly two values of x. (D) $f(x) = 0$ has a root in interval (0, 1).

9. Let $P(x) = x^{32} - x^{25} + x^{18} - x^{11} + x^4 - x^3 + 1$. Which of the following are **CORRECT** ?

(A) Number of real roots of $P(x) = 0$ are zero.
(B) Number of imaginary roots of $P(x) = 0$ are 32.
(C) Number of negative roots of $P(x) = 0$ are zero.
(D) Number of imaginary roots of $P(x) + P(-x) = 0$ are 32.

10. If α, β are the real and distinct roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always (given $\alpha \neq -\beta$)

(A) two real roots (B) two negative roots
(C) two positive roots (D) one positive root and one negative root

11. $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is $(a, b, c, d \in R)$

(A) $-d/a$ (B) d/a (C) $(b - a)/a$ (D) $(a - b)/a$

12. If $-5 + i\beta$, $-5 + i\gamma$ (where $\beta^2 \neq \gamma^2$; $\beta, \gamma \in R$ and $i^2 = -1$) are roots of $x^3 + 15x^2 + cx + 860 = 0$, $c \in R$, then

(A) $c = 222$
(B) all the three roots are imaginary
(C) two roots are imaginary but not complex conjugate of each other.
(D) $-5 + 7i\sqrt{3}$, $-5 - 7i\sqrt{3}$ are imaginary roots.

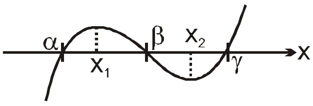
13. Let $f(x) = ax^2 + bx + c > 0$, $\forall x \in R$ or $f(x) < 0$, $\forall x \in R$. Which of the following is/are **CORRECT** ?

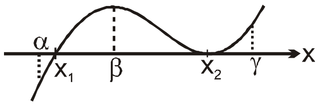
(A) If $a + b + c > 0$ then $f(x) > 0$, $\forall x \in R$ (B) If $a + c < b$ then $f(x) < 0$, $\forall x \in R$
(C) If $a + 4c > 2b$ then $f(x) < 0$, $\forall x \in R$ (D) $ac > 0$.

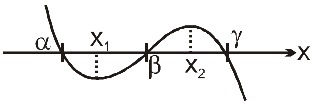
14. Let $x_1 < \alpha < \beta < \gamma < x_4$, $x_1 < x_2 < x_3$. If f(x) is a cubic polynomial with real coefficients such that $(f(\alpha))^2 + (f(\beta))^2 + (f(\gamma))^2 = 0$, $f(x_1)f(x_2) < 0$, $f(x_2)f(x_3) < 0$ and $f(x_1)f(x_3) > 0$ then which of the following are **CORRECT** ?

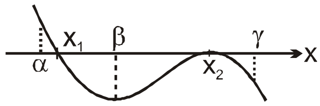
(A) $\alpha \in (x_1, x_2)$, $\beta \in (x_2, x_3)$ and $\gamma \in (x_3, x_4)$ (B) $\alpha \in (x_1, x_3)$, $\beta, \gamma \in (x_3, x_4)$
(C) $\alpha, \beta \in (x_1, x_2)$ and $\gamma \in (x_4, \infty)$ (D) $\alpha \in (x_1, x_3)$, $\beta \in (x_2, x_3)$ and $\gamma \in (x_2, x_4)$

15. If f(x) is cubic polynomial with real coefficients, $\alpha < \beta < \gamma$ and $x_1 < x_2$ be such that $f(\alpha) = f(\beta) = f(\gamma) = f'(x_1) = f'(x_2) = 0$ then possible graph of y = f(x) is (assuming y-axis vertical)

(A) 

(B) 

(C) 

(D) 

16. Let $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$, then f(x) = 0 has
(A) exactly one real root in (2, 3)
(B) exactly one real root in (3, 4)
(C) 3 different roots
(D) atleast one negative root

17. If the quadratic equations $ax^2 + bx + c = 0$ ($a, b, c \in R$, $a \neq 0$) and $x^2 + 4x + 5 = 0$ have a common root, then a, b, c must satisfy the relations:
(A) $a > b > c$
(B) $a < b < c$
(C) $a = k$; $b = 4k$; $c = 5k$ ($k \in R$, $k \neq 0$)
(D) $b^2 - 4ac$ is negative.

18. If the quadratic equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then the equation containing their other roots is/are :
(A) $x^2 + a(b + c)x - a^2bc = 0$
(B) $x^2 - a(b + c)x + a^2bc = 0$
(C) $a(b + c)x^2 - (b + c)x + abc = 0$
(D) $a(b + c)x^2 + (b + c)x - abc = 0$

19. Consider the following statements.
S_1 : The equation $2x^2 + 3x + 1 = 0$ has irrational roots.
S_2 : If $a < b < c < d$, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are real and distinct.
S_3 : If $x^2 + 3x + 5 = 0$ and $ax^2 + bx + c = 0$ have a common root and $a, b, c \in N$, then the minimum value of (a + b + c) is 10.
S_4 : The value of the biquadratic expression $x^4 - 8x^3 + 18x^2 - 8x + 2$, when $x = 2 + \sqrt{3}$, is 1
Which of the following are **CORRECT** ?
(A) S_2 and S_4 are true.
(B) S_1 and S_3 are false.
(C) S_1 and S_2 are true.
(D) S_3 and S_4 are false.

20. If the equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ & $x^2 + (a + b)x + 36 = 0$ have a common positive root, then which of the following are true ?
(A) ab = 56
(B) common positive root is 3
(C) sum of uncommon roots is 21.
(D) a + b = 15.

21. If $x^2 + \lambda x + 1 = 0$, $\lambda \in (-2, 2)$ and $4x^3 + 3x + 2c = 0$ have common root then $c + \lambda$ can be

(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) 0
(D) $\frac{3}{2}$

22_. Let quadratic equation p(x) = 0 (where $p(x) = x^2 + bx + c$) and equation p(p(p(x))) = 0 has a common root, then which of the following statement is/are correct.
(A) If $b, c \in R$, then $b^2 - 4c \geq 0$
(B) If P(0) = 1, then p(1) = 0
(C) equations p(p(p(x))) = 0 and p(p(p(p(p(x))))) = 0 has at least two common root.
(D) zero is root of equation p(p(p(p(p(p(x)))))) = 0

PART - IV : COMPREHENSION

Comprehension # 1 (Q. No. 1 & 2)

If x, y $\in$ R then some problems can be solved by direct observing extreme cases

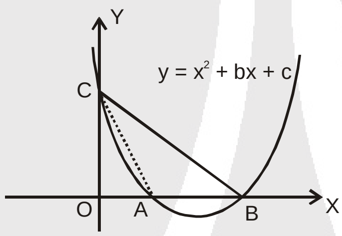
e.g. (i) $(x - 3)^2 + (y - 2)^2 = 0$ is possible only for x = 3 and y = 2
(ii) if $x \geq 3$, $y \geq 2$ and $xy \leq 6$ then x = 3 & y = 2

1. The least value of expression $x^2 + 2xy + 2y^2 + 4y + 7$ is :
(A) 1 (B) 2 (C) 3 (D) 4

2. Let $P(x) = 4x^2 + 6x + 4$ and $Q(y) = 4y^2 - 12y + 25$. If x, y satisfy equation P(x).Q(y) = 28, then the value of 11y – 26x is -
(A) 6 (B) 36 (C) 8 (D) 42

Comprehension # 2 (Q. No. 3 & 4)

In the given figure ΔOBC is an isosceles right triangle in which AC is a median, then answer the following questions :



3. Roots of y = 0 are
(A) {2, 1} (B) {4, 2} (C) {1, 1/2} (D) {8, 4}

4. The equation whose roots are $(\alpha + \beta)$ & $(\alpha - \beta)$, where α, β $(\alpha > \beta)$ are roots obtained in previous question, is
(A) $x^2 - 4x + 3 = 0$ (B) $x^2 - 8x + 12 = 0$ (C) $4x^2 - 8x + 3 = 0$ (D) $x^2 - 16x + 48 = 0$

Comprehension # 3 (Q. No. 5 to 7)

Consider the equation $x^4 - \lambda x^2 + 9 = 0$. This can be solved by substituting $x^2 = t$ such equations are called as pseudo quadratic equations.

5. If the equation has four real and distinct roots, then λ lies in the interval
(A) $(-\infty, -6) \cup (6, \infty)$ (B) $(0, \infty)$ (C) $(6, \infty)$ (D) $(-\infty, -6)$

6. If the equation has no real root, then λ lies in the interval
(A) $(-\infty, 0)$ (B) $(-\infty, 6)$ (C) $(6, \infty)$ (D) $(0, \infty)$

7. If the equation has only two real roots, then set of values of λ is
(A) $(-\infty, -6)$ (B) $(-6, 6)$ (C) $\{6\}$ (D) ϕ

Comprehension # 4 (Q. No. 8 to 10)

To solve equation of type,
$ax^{2m} + bx^{2m-1} + cx^{2m-2} + \ldots\ldots\ldots + kx^m + \ldots\ldots\ldots + cx^2 + bx + a = 0,$ $(a \neq 0) \rightarrow$ (I)
divide by x^m and rearrange terms to obtain

$$a\left(x^m + \frac{1}{x^m}\right) + b\left(x^{m-1} + \frac{1}{x^{m-1}}\right) + c\left(x^{m-2} + \frac{1}{x^{m-2}}\right) + \ldots\ldots\ldots + k = 0$$

Substitutions like

$t = x + \dfrac{1}{x}$ or $t = x - \dfrac{1}{x}$ helps transforming equation into a reduced degree equation.

8.✍ Roots of equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ are

(A) $2 \pm \sqrt{3}$, $3 \pm \sqrt{2}$ (B) $2 \pm \sqrt{3}$, $3 \pm 2\sqrt{2}$
(C) $3 \pm \sqrt{2}$, $3 \pm 2\sqrt{2}$ (D) $8 \pm \sqrt{3}$, $3 \pm \sqrt{2}$

9.✍ Roots of equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$ are

(A) $1, \frac{3 \pm \sqrt{5}}{2}, \frac{1 \pm i\sqrt{3}}{2}$ (B) $1, \frac{5 \pm \sqrt{3}}{2}, \frac{3 \pm i}{2}$
(C) $1, \frac{3 \pm \sqrt{5}}{2}, \frac{3 \pm i}{2}$ (D) $1, \frac{5 \pm \sqrt{3}}{2}, \frac{1 \pm i\sqrt{3}}{2}$

10.✍ Roots of equation $x^6 - 4x^4 + 4x^2 - 1 = 0$ are

(A) $\pm 1, \frac{1 \pm i\sqrt{5}}{2}, \frac{-1 \pm \sqrt{5}}{2}$ (B) $\pm 1, \frac{1 \pm \sqrt{5}}{2}, \frac{-1 \pm i\sqrt{5}}{2}$
(C) $\pm 1, \frac{1 \pm \sqrt{5}}{2}, \frac{-1 \pm \sqrt{5}}{2}$ (D) $\pm 1, \frac{-1 \pm \sqrt{5}}{2}, \frac{-1 \pm i\sqrt{5}}{2}$.

Exercise-3

✍ Marked questions are recommended for Revision.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[IIT-JEE 2010, Paper-1, (3, –1)/ 84]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ (B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ (D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

2. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) 1 (B) 2 (C) 3 (D) 4

3. A value of b for which the equations **[IIT-JEE 2011, Paper-2, (3, –1), 80]**
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

4. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

(A) only purely imaginary roots (B) all real roots
(C) two real and two purely imaginary roots (D) neither real nor purely imaginary roots

5*. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **[JEE (Advanced) 2015, P-2 (4, –2)/ 80]**

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$ (C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

6. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals
[JEE (Advanced) 2016, Paper-1, (3, –1)/62]
(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Comprehension (Q-7 & 8)

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \neq \beta$.
For n = 0,1,2,...., let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b.

7. a_{12} = **[JEE(Advanced) 2017, Paper-2,(3, 0)/61]**
(A) $a_{11} + 2a_{10}$ (B) $2a_{11} + a_{10}$ (C) $a_{11} - a_{10}$ (D) $a_{11} + a_{10}$

8. If $a_4 = 28$, then p + 2q = **[JEE(Advanced) 2017, Paper-2,(3, 0)/61]**
(A) 14 (B) 7 (C) 21 (D) 12

9*. Let α and β be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$. For all postive integers n. define
$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$
$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$
the which of the follwing options is/are correct ? **[JEE(Advanced) 2019, Paper-1,(4, –1)/62]**
(1) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$ (2) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$
(3) $a_1 + a_2 + ...a_n = a_{n+2} - 1$ for all $n \geq 1$ (4) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Sachin and Rahul attempted to solve a quadratic equaiton. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are : **[AIEEE- 2011, II, (4, –1), 120]**
(1) 6, 1 (2) 4, 3 (3) –6 , –1 (4) –4 , –3

2. Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of p(2) is : **[AIEEE- 2011, II, (4, –1), 120]**
(1) 3 (2) 9 (3) 6 (4) 18

3. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[AIEEE- 2012 (4, –1), 120]**
(1) infinite number of real roots (2) no real roots
(3) exactly one real root (4) exactly four real roots

4. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a,b,c \in R$, have a common root, then a : b : c is
[AIEEE - 2013, (4, –1), 120]
(1) 1 : 2 : 3 (2) 3 : 2 : 1 (3) 1 : 3 : 2 (4) 3 : 1 : 2

5. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no intgeral solution, then all possible values of a lie in the interval :
[JEE(Main) 2014, (4, – 1), 120]
(1) (–2, –1) (2) $(-\infty, -2) \cup (2, \infty)$ (3) $(-1, 0) \cup (0, 1)$ (4) (1, 2)

6. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q ,r are in the A.P. and $\frac{1}{\alpha}+\frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$ (3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

7. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[JEE(Main) 2015, (4, – 1), 120]**

(1) 6 (2) – 6 (3) 3 (4) –3

8. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is : **[JEE(Main) 2019, Online (09-01-19),P-2 (4, – 1), 120]**

(1) 3 (2) 4 (3) 5 (4) 2

9. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m - 4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is : **[JEE(Main) 2019, Online (12-01-19),P-1 (4, – 1), 120]**

(1) $-2 + \sqrt{2}$ (2) $4 - 3\sqrt{2}$ (3) $2 - \sqrt{3}$ (4) $4 - 2\sqrt{3}$

10. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is :

[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120]

(1) 3 (2) 4 (3) 2 (4) 5

11. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then

$\lim\limits_{n\to\infty} \sum\limits_{r=1}^{n} \alpha^r \lim\limits_{n\to\infty} \sum\limits_{r=1}^{n} \beta^r$ is equal to : **[JEE(Main) 2019, Online (12-04-19),P-1 (4, – 1), 120]**

(1) $\frac{29}{358}$ (2) $\frac{21}{346}$ (3) $\frac{7}{116}$ (4) $\frac{1}{12}$

12. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to -

[JEE(Main) 2019, Online (12-04-19),P-2 (4, – 1), 120]

(1) 0 (2) $\alpha\gamma$ (3) $\beta\gamma$ (4) $\alpha\beta$

13. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which of the following statements is not true? **[JEE(Main) 2020, Online (07-01-20),P-2 (4, –1), 120]**

(1) $p_5 = p_2 . p_3$ (2) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(3) $p_3 = p_5 - p_4$ (4) $p_5 = 11$

14. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is :

[JEE(Main) 2020, Online (09-01-20),P-1 (4, –1), 120]

(1) 3 (2) 1 (3) 4 (4) 2

15. Let $a, b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to:

[JEE(Main) 2020, Online (09-01-20),P-2 (4, –1), 120]

(1) 25 (2) 26 (3) 24 (4) 28

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. a = 2; No real value of x. **A-2.** (i) $-\frac{7}{4}$ (ii) $-\frac{7}{8}$

A-3. (i) $ac\,x^2 + b(a + c)\,x + (a + c)^2 = 0$ (ii) $a^2 x^2 + (2ac - 4a^2 - b^2)\,x + 2b^2 + (c - 2a)^2 = 0$

A-4. $3x^2 - 19x + 3 = 0$. **A-5.** 8, 3

A-6. (i) 4 (ii) 72 (iii) 2

A-7. $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$

A-10. 2 **A-11.** 11

Section (B) :

B-2. $-\frac{(r+1)^3}{r^2}$

B-3. (i) roots are $\frac{3}{4}, \frac{3}{2}, \frac{-5}{3}$, $\lambda = 45$ or $\frac{-1}{2}, -1, \frac{25}{12}$, $\lambda = -25$.

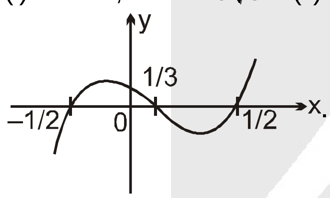
(ii) roots are $\frac{-4}{3}, -\frac{3}{2}, \frac{-5}{3}$, $\lambda = 121$

B-4. $x^3 - 15x^2 + 67x - 77 = 0$. **B-5.** -3 **B-6.** $\mathbf{\frac{1}{2}, \frac{1}{2}, -6}$

Section (C) :

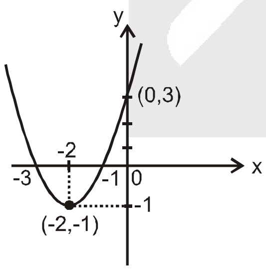
C-1. $(-4, 7)$ **C-3.** $3 \pm 2\sqrt{2}$

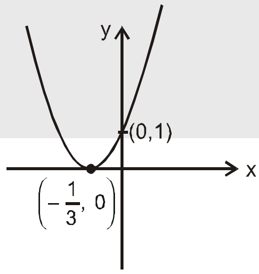
C-7. (i) $4,\ -2 \pm i5\sqrt{3}$ (ii) 3 or 4 **C-8.** $-1 \pm \sqrt{2}, -1 \pm \sqrt{-1}$

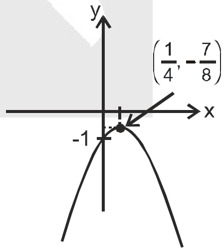
C-9. 

Two positive roots.

Section (D) :

D-1. (i) 

(ii) 

(iii) 

D-2. (i) $(-\infty, 4]$ (ii) $[2, 6]$ (iii) $[3, 6)$

D-3. (i) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (ii) $\left(-\infty, \frac{-4}{5}\right] \cup (1, \infty)$ **D-4.** $\left(-\infty, -\frac{1}{2}\right)$

D-5. (i) $a > 1$ (ii) $a \in \phi$.

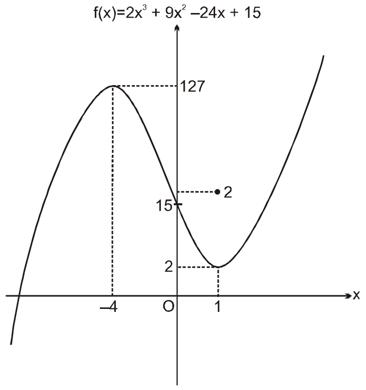
Section (E) :

E-2. $K \in (-2, 3)$ **E-3.** $a \in (-2, 2)$ **E-4.** $a \in (1, 5) - \{3\}$ **E-5.** $6 < K < 6.75$

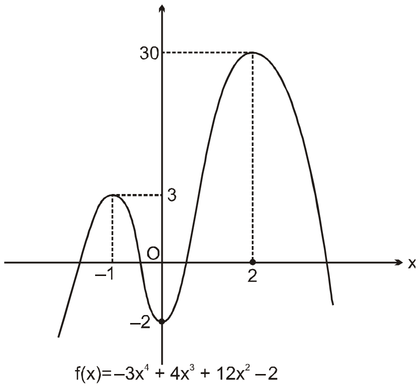
Section (F) :

F-2. a = 0, 24 **F-3.** 3

F-5. (i)



(ii)



F-6. (i) $k \in [-2,2]$
(ii) $k \in (-\infty,-2) \cup (2, \infty)$

PART - II

Section (A) :

A-1. (B) **A-2.** (C) **A-3.** (A) **A-4.** (C) **A-5.** (A)

Section (B) :

B-1. (C) **B-2.** (C) **B-3.** (B) **B-4.** (A) **B-5.** (C)

Section (C) :

C-1. (B) **C-2.** (C) **C-3.** (A) **C-4.** (A) **C-5.** (A) **C-6.** (C)

Section (D) :

D-1. (B) **D-2.** (B) **D-3.** (B) **D-4.** (B) **D-5.** (A) **D-6.** (C) **D-7.** (C)
D-8. (D)

Section (E) :

E-1. (D) **E-2.** (B) **E-3.** (D) **E-4.** (D)

Section (F) :

F-1. (A) **F-2.** (C) **F-3.** (A) **F-4.** (C) **F-5.** (D)

PART - III

1. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (s)

2. (A $\rightarrow$ r); (B $\rightarrow$ p,q,s); (C $\rightarrow$ s); (D $\rightarrow$ p,q,r)

3. (A) q, s, t (B) p, t (C) r (D) q, s.

EXERCISE - 2

PART - I

1.	(B)	**2.**	(B)	**3.**	(D)	**4.**	(B)	**5.**	(A)	**6.**	(A)	**7.**	(A)
8.	(B)	**9.**	(A)	**10.**	(D)	**11.**	(B)	**12.**	(B)	**13.**	(A)	**14.**	(C)

PART - II

1.	29.00	**2.**	08.00	**3.**	01.00	**4.**	68.13	**5.**	01.12	**6.**	13.20	**7.**	13.00
8.	39.30	**9.**	01.00	**10.**	02.25	**11.**	11.33	**12.**	32.00	**13.**	01.00	**14.**	09.00
15.	01.00	**16.**	03.50	**17.**	08.12								

PART - III

1.	(ACD)	**2.**	(BCD)	**3.**	(BC)	**4.**	(AC)	**5.**	(BCD	**6.**	(BC)	**7.**	(ABD)
8.	(ABCD)	**9.**	(ABCD)	**10.**	(AD)	**11.**	(AD)	**12.**	(AD)	**13.**	(ABD)	**14.**	(AD)
15.	(AC)	**16.**	(AB)	**17.**	(CD)	**18.**	(BD)	**19**.	(AB)	**20.**	(ABC)	**21.**	(AB)
22.	(ABCD)												

PART - IV

1.	(C)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(C)	**6.**	(B)	**7.**	(D)
8.	(B)	**9.**	(A)	**10.**	(C)								

EXERCISE - 3

PART - I

1.	(B)	**2.**	(C)	**3.**	(B)	**4.**	(D)	**5.**	(A, D)	**6.**	(C)
7.	(D)	**8.**	(D)	**9*.**	(A,B,C)						

PART - II

1.	(1)	**2.**	(4)	**3.**	(2)	**4.**	(1)	**5.**	(3)	**6.**	(2)	**7.**	(3)
8.	(1)	**9.**	(2)	**10.**	(2)	**11.**	(1)	**12.**	(3)	**13.**	(1)	**14.**	(2)
15.	(1)												

High Level Problems (HLP)

1. Find the number of values of x satisfying the relation

$$\alpha_1^3\left(\frac{\prod_{i=2}^{n}(x-\alpha_i)}{\prod_{i=2}^{n}(\alpha_1-\alpha_i)}\right)+\sum_{j=2}^{n-1}\left(\left(\frac{\prod_{i=1}^{j-1}(x-\alpha_i)\prod_{i=j+1}^{n}(x-\alpha_i)}{\prod_{i=1}^{j-1}(\alpha_j-\alpha_i)\prod_{i=j+1}^{n}(\alpha_j-\alpha_i)}\right)\alpha_j^3\right)+\left(\frac{\prod_{i=1}^{n-1}(x-\alpha_i)}{\prod_{i=1}^{n-1}(\alpha_n-\alpha_i)}\right)\alpha_n^3-x^3=0 \text{ (where } n\geq 5).$$

2. Prove that roots of $a^2x^2 + (b^2 + a^2 - c^2)\,x + b^2 = 0$ are not real, if $a + b > c$ and $|a - b| < c$. (where a, b, c are positive real numbers)

3. Solve the inequality, $\frac{1}{x-1}-\frac{4}{x-2}+\frac{4}{x-3}-\frac{1}{x-4}<\frac{1}{30}$.

4. If three real and distinct numbers a, b, c are in G.P. (i.e., $b^2 = ac$) and $a + b + c = x\,b$, then prove that $x < -1$ or $x > 3$.

5. If $V_n = \alpha^n + \beta^n$, where α, β are roots of equation $x^2 + x - 1 = 0$. Then prove that $V_n + V_{n-3} = 2\,V_{n-2}$ and hence evaluate V_7 (n is a whole number)

6. Find all 'm' for which $f(x) \equiv x^2 - (m - 3)\,x + m > 0$ for all values of 'x' in [1, 2].

7. Find the values of a, for which the quadratic expression $ax^2 + (a - 2)\,x - 2$ is negative for exactly two integral values of x.

8. Find the number of real roots of $\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$

9. If α, β are roots of the equation $x^2 - 34x + 1 = 0$, evaluate $\sqrt[4]{\alpha}-\sqrt[4]{\beta}$, where $\sqrt[4]{\;.\;}$ denotes the principal value.

10. Find the values of 'a' for which the equation

$\left(x^2+x+2\right)^2-(a-3)\left(x^2+x+2\right)\left(x^2+x+1\right)+(a-4)\left(x^2+x+1\right)^2=0$ has atleast one real root.

11. Show that the quadratic equation $x^2 + 7x - 14(q^2 + 1) = 0$ where q is an integer, has no integral roots.

12. Find the integral values of 'a' for which the equation $x^4-\left(a^2-5a+6\right)x^2-\left(a^2-3a+2\right)=0$ has only real roots.

13. If α, β; β, γ and γ, α are the roots of $a_ix^2+b_ix+c_i=0$; i = 1, 2, 3 then show that

$$(\alpha+\beta+\gamma)+(\alpha\beta+\beta\gamma+\alpha\gamma)+\alpha\beta\gamma=\pm\left\{\prod_{i=1}^{3}\left(\frac{a_i-b_i+c_i}{a_i}\right)\right\}^{1/2}-1$$

14. Suppose that $a_1 > a_2 > a_3 > a_4 > a_5 > a_6$ and

$p = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

$q = a_1a_3 + a_3a_5 + a_5a_1 + a_2a_4 + a_4a_6 + a_6a_2$

$r = a_1a_3a_5 + a_2a_4a_6$,

then show that roots of the equation $2x^3 - px^2 + qx - r = 0$ are real.

15. If $\beta + \cos^2\alpha$, $\beta + \sin^2\alpha$ are the roots of $x^2 + 2bx + c = 0$ and $\gamma + \cos^4\alpha$, $\gamma + \sin^4\alpha$ are the roots of $X^2 + 2BX + C = 0$, then prove that $b^2 - B^2 = c - C$.

16. Find the set of values of 'a' if $(x^2 + x)^2 + a(x^2 + x) + 4 = 0$ has
(i) all four real & distinct roots.
(ii) four roots in which only two roots are real and distinct.
(iii) all four imaginary roots.
(iv) four real roots in which only two are equal.

17. $f(x) = x^2 + bx + c$, where $b,c \in R$, if f(x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$ then find f(x).

18. Let $ax^4 + bx^3 + x^2 + (3-a)x + 3 = 0$ and $x^2 + (2-a)x + 3 = 0$ have common roots. If $a \in (-1,5)$ then find $|a+12b|$

19. How many quadratic equations are there which are unchanged by squaring their roots ?

20. Let $P(x) = x^5 + x^2 + 1$ have zeros $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ and $Q(x) = x^2 - 2$, then find

(i) $\prod_{i=1}^{5} Q(\alpha_i)$ (ii) $\sum_{i=1}^{5} Q(\alpha_i)$ (iii) $\sum_{1 \le i < j \le 5} \sum Q(\alpha_i)\, Q(\alpha_j)$ (iv) $\sum_{i=1}^{5} Q^2(\alpha_i)$

21. If a, b, c are non–zero, unequal rational numbers then prove that the roots of the equation $(abc^2)x^2 + 3a^2cx + b^2cx - 6a^2 - ab + 2b^2 = 0$ are rational.

22. If a, b, c represents sides of a Δ then prove that equation $x^2 - (a^2 + b^2 + c^2)x + a^2b^2 + b^2c^2 + c^2a^2 = 0$ has imaginary roots.

23. If x_1 is a root of $ax^2 + bx + c = 0$, x_2 is a root of $-ax^2 + bx + c = 0$ where $0 < x_1 < x_2$, show that the equation $ax^2 + 2bx + 2c = 0$ has a root x_3 satisfying $0 < x_1 < x_3 < x_2$.

24. Find the number of positive real roots of $x^4 - 4x - 1 = 0$.

25. If $(1 + k)\tan^2 x - 4\tan x - 1 + k = 0$ has real roots $\tan x_1$ and $\tan x_2$, where $\tan x_1 \ne \tan x_2$, then find k.

26. Let Δ^2 be the discriminant and α, β be the roots of the equation $ax^2 + bx + c = 0$. Then find equation whose roots are $2a\alpha + \Delta$ and $2a\beta - \Delta$.

27. Prove that $\frac{\pi^e}{x-e} + \frac{e^\pi}{x-\pi} + \frac{\pi^\pi + e^e}{x-\pi-e} = 0$ has one real root in (e, π) and other in $(\pi, \pi + e)$.

28. If α, β^2 are integers, β^2 is non-zero multiple of 3 and $\alpha + i\beta$, -2α are roots of $x^3 + ax^2 + bx - 316 = 0$, $a, b, \beta \in R$, then find a, b.

29. Let polynomial $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ have integral coefficient (where $a > 0$) If there exist four distinct integer $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ ($\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4$) such that $f(\alpha_1) = f(\alpha_2) = f(\alpha_3) = f(\alpha_4) = 5$ and equation $f(x) = 9$ has atleast one integeral roots then find

(i) $f\left(\frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{4}\right)$ (ii) $f'\left(\frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{4}\right)$

(iii) Range of f(x) in $[\alpha_2, \alpha_3]$

(iv) Difference of largest and smallest root of equation $f(x) = 9$

30. If x and y both are non-negative integral values for which $(xy - 7)^2 = x^2 + y^2$, then find the sum of all possible values of x.

31. Find the set of all real values of λ such that the root of the equation
$x^2 + 2(a + b + c)x + 3\lambda (ab + bc + ca) = 0$ are always real for any choice of a, b, c (where a, b, c represents sides of scalene triangle).

(A) $\left(-\infty, \frac{4}{3}\right)$ (B) $\left(\frac{4}{3}, \infty\right)$ (C) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{5}{3}\right)$

32_. Let $P(x) = x^2 + bx + c(b, c \in R)$, then which of the following statement implies that $P(P(x)) = 0$ has atleast one negative root.

(A) $P(x) = 0$ has root of opposite sign
(B) $P(x) = 0$ has both roots positive
(C) $P(x) = 0$ has both roots negative
(D) $\left(c - \frac{b^2}{4}\right)^2 + b\left(c - \frac{b^2}{4}\right) + c < 0$ & $b > 0$

Answers

1.	Infinite	**3.**	$(-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$
5.	–29	**6.**	$(-\infty, 10)$
7.	[1, 2)	**8.**	0
9.	± 2	**10.**	$5 < a \le \frac{19}{3}$

12. $a \in \{1, 2\}$

16. (i) $a \in (-\infty, -4)$ (ii) $a \in \left(\frac{65}{4}, \infty\right)$ (iii) $a \in \left(-4, \frac{65}{4}\right)$ (iv) $a \in \phi$

17. $x^2 - 2x + 5$ **18.** 3 **19.** 4

20. (i) – 23 (ii) – 10 (iii) 40 (iv) 20

24. 1 **25.** $\left(-\sqrt{5}, -1\right) \cup \left(-1, \sqrt{5}\right)$

26. $x^2 + 2b\,x + b^2 = 0$ or $x^2 + 2bx - 3b^2 + 16\,ac = 0$

28. a = 0, b = 63

29. (i) 9 (ii) 0 (iii) [5, 9] (iv) $2\sqrt{5}$

30. 14 **31.** (A) **32.** (AD)

HINTS & SOLUTIONS

TOPIC : QUADRATIC EQUATION

EXERCISE # 1

PART–1

A-1. $a^2 - a - 2 = 0$, $a^2 - 4 = 0$, $a^2 - 3a + 2 = 0 \Rightarrow a = 2, -1$ and $a = \pm 2$ and $a = 1, 2 \Rightarrow a = 2$

Now $(x^2 + x + 1)\, a^2 - (x^2 + 3)\, a - (2x^2 + 4x - 2) = 0$ will be an identity if $x^2 + x + 1 = 0$ & $x^2 + 3 = 0$ & $2x^2 + 4x - 2 = 0$ which is not possible.

A-2. (i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{-3}{2}\right)^2 - 2(2) = \frac{-7}{4}$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = -\frac{7}{8}$

A-3. $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

(i) $\alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta} = \frac{-b}{a} + \frac{-b/a}{c/a} = -\left(\frac{b}{a} + \frac{b}{c}\right) = -b\frac{(a+c)}{ac}$

and $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2 = \frac{c}{a} + \frac{a}{c} + 2 = \frac{(a+c)^2}{ac}$

$\therefore$ equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ is

$\therefore$ $\alpha + \frac{1}{\beta}\,\beta + \frac{1}{\alpha} \Rightarrow acx^2 + b(a + c)x + (a + c)^2 = 0$

(ii) $\alpha^2 + 2 + \beta^2 + 2 = (\alpha + \beta)^2 - 2\alpha\beta + 4 = \frac{b^2}{a^2} - \frac{2ac}{a^2} + 4 = \frac{4a^2 + b^2 - 2ac}{a^2}$

and $(\alpha^2 + 2)(\beta^2 + 2) = \alpha^2\beta^2 + 2(\alpha^2 + \beta^2) + 4 = \frac{c^2}{a^2} + \frac{2(b^2 - 2ac)}{a^2} + 4$

$\therefore$ equation whose roots are $\alpha^2 + 2$ & $\beta^2 + 2$ is

$a^2 x^2 + (2ac - b^2 - 4a^2)x + 2b^2 + 4a^2 + c^2 - 4ac = 0$

$\Rightarrow a^2 x^2 + (2ac - b^2 - 4a^2)x + 2b^2 + (2a - c)^2 = 0$

A-4. given $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$

$\Rightarrow$ α & β are the roots of $x^2 - 5x + 3 = 0$

$\therefore$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{25 - 6}{3} = \frac{19}{3}$ and $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$\therefore$ equation have $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ as its roots is $3x^2 - 19x + 3 = 0$

A-5. $x^2 + px + q = 0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \beta\end{smallmatrix}$ $\Rightarrow p = -11,\ q = 24$

then correct equation will be $x^2 - 11x + 24 = 0$

$x^2 - 11x + 24 = 0$

$\Rightarrow$ $(x - 8)(x - 3) = 0$ $\Rightarrow x = 3, 8$

A-6. **(i)** $E = 2x^3 + 2x^2 - 7x + 72$

Given, $x = \frac{3 + 5i}{2}$

$\Rightarrow$ $2x - 3 = 5i$

$\Rightarrow$ $4x^2 + 9 - 12x = -25$

$\Rightarrow$ $4x^2 - 12x + 34 = 0$

$\Rightarrow$ $2x^2 - 6x + 17 = 0$ ....(i)

Given expression can be written as

$E = (2x^2 - 6x + 17)(x + 4) + 4 = 4$ (using (i))

(ii) $\left(x + \frac{1}{2}\right) = \frac{\sqrt{15}}{2} \Rightarrow x^2 + x + \frac{1}{4} = \frac{15}{4} \Rightarrow x^2 + x = \frac{14}{4} \Rightarrow x^2 + x = \frac{7}{2}$

$\therefore \quad 2x^3 + 2x^2 - 7x + 72 = 2x(x^2 + x) - 7x + 72 = 2x\left(\frac{7}{2}\right) - 7x + 72 = 7x - 7x + 72 = 72.$

(iii) $2^x = y \Rightarrow y^2 + 2^2 y - 32 = 0 \Rightarrow y^2 + 8y - 4y - 32 = 0$

$\Rightarrow y = 4 = 2^x \quad \because 2^x \neq -8 \Rightarrow x = 2.$

A-7. $\because \quad ax^2 + bx + c = 0 \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \Rightarrow \alpha + \beta = -\frac{b}{a} \Rightarrow \alpha\beta = \frac{c}{a}$

$\because$ Let $a^3 x^2 + (a\,b\,c)\,x + c^3 = 0 \begin{smallmatrix} \gamma \\ \delta \end{smallmatrix}$

$\therefore \quad \gamma + \delta = -\frac{abc}{a^3} = \left(-\frac{b}{a}\right)\left(\frac{c}{a}\right) = (\alpha\,\beta)(\alpha + \beta) = \alpha^2\beta + \alpha\beta^2$ v ....(i)

$\because \quad \gamma\,\delta = \left(\frac{c}{a}\right)^3 = (\alpha\beta)^3 = (\alpha^2\beta)(\alpha\beta^2)$ ....(ii)

From (i) and (ii) we can say that $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ and $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$

A-8. $\alpha + \beta = p,\ \alpha\beta = q \Rightarrow (\alpha - 2)(\beta + 2) = r \Rightarrow \alpha\beta + 2\alpha - 2\beta - 4 = r$

$q + 2(\alpha - \beta) - 4 = r \Rightarrow 2\alpha - 2\beta = r + 4 - q \Rightarrow 2\alpha + 2\beta = 2p$

$4\alpha = r + 4 - q + 2p \Rightarrow 4\beta = 2p - (r + 4 - q) \Rightarrow 16\alpha\beta = 4p^2 - (r + 4 - q)^2$

$16q + (r + 4 - q)^2 = 4p^2.$

A-9. $\alpha \,.\, \alpha^n = \frac{c}{a} \Rightarrow \alpha = \left(\frac{c}{a}\right)^{\frac{1}{n+1}} \Rightarrow \alpha + \alpha^n = -\frac{b}{a} \Rightarrow \left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = -\frac{b}{a}$

$a^{1-\frac{1}{n+1}} \,.\, c^{\frac{1}{n+1}} + c^{\frac{n}{n+1}} .\ a^{1-\frac{n}{n+1}} + b = 0 \Rightarrow a^{\frac{n}{n+1}} \,.\, c^{\frac{1}{n+1}} + a^{\frac{1}{n+1}} .\ c^{\frac{n}{n+1}} + b = 0$

$\left(a^n\ c\right)^{\frac{1}{n+1}} + \left(a\ c^n\right)^{\frac{1}{n+1}} + b = 0$ Proved.

A-10. $S = \frac{-(2a+3)}{a+1} = -1 \Rightarrow 2a + 3 = a + 1 \Rightarrow a = -2;\ p = \frac{3a+4}{a+1} = \frac{-6+4}{-2+1} = 2$

A-11. $2x^2 + 6x + a = 0$

$\because$ Its roots are $\alpha, \beta \Rightarrow \alpha + \beta = -3\ \&\ \alpha\beta = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$

$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} < 2 \Rightarrow \frac{9-a}{a} < 1$

$\Rightarrow \frac{2a-9}{a} > 0 \Rightarrow a \in (-\infty, 0) \cup \left(\frac{9}{2}, \infty\right) \Rightarrow 2a = 11$ is least prime.

B-1. Let 3rd root be γ then $\alpha\beta\gamma = -r$ But $\alpha\beta = -1$ (given) $\Rightarrow \gamma = r$

substituting $x = \gamma = r$ in the given equation we get $r^2 + pr + q + 1 = 0$.

B-2. $x^3 + px^2 + qx + r \begin{smallmatrix} \alpha \\ \beta \\ \gamma \end{smallmatrix} \Rightarrow \alpha\beta\gamma = -r \Rightarrow \left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right)$

$= \left(\alpha + \frac{\alpha}{r}\right)\left(\beta + \frac{\beta}{r}\right)\left(\gamma + \frac{\gamma}{r}\right) = \alpha\beta\gamma\left(1 + \frac{1}{r}\right)^3 = -r\ \frac{(r+1)^3}{r^3} = -\frac{(r+1)^3}{r^2}$ **Ans.**

B-3. **(i)** Let roots be $\alpha, 2\alpha, \beta \Rightarrow 3\alpha + \beta = \frac{14}{24}$, $2\alpha^2 + 3\alpha\beta = \frac{-63}{24}$, $2\alpha^2\beta = \frac{-\lambda}{24} \Rightarrow \beta = \frac{7}{12} - 3\alpha$

$2\alpha^2 + 3\alpha\left(\frac{7}{12} - 3\alpha\right) = \frac{-21}{8} \Rightarrow 2\alpha^2 + \frac{7\alpha}{4} - 9\alpha^2 = \frac{-21}{8} \Rightarrow 0 = 7\alpha^2 - \frac{7\alpha}{4} - \frac{21}{8}$

$\alpha^2 - \frac{\alpha}{4} - \frac{3}{8} = 0 \Rightarrow \quad 8\alpha^2 - 2\alpha - 3 = 0 \Rightarrow \quad \alpha = \frac{3}{4}$ or $\frac{-1}{2}$

$\alpha = \frac{3}{4} \quad \Rightarrow$ roots are $\frac{3}{4}, \frac{3}{2}, \frac{-5}{3}$ and $\lambda = 45 \Rightarrow \quad \alpha = \frac{-1}{2}$

$\Rightarrow$ roots are $\frac{-1}{2}, -1, \frac{25}{12}$ and $\lambda = -25$

(ii) α, β, γ be roots.

$\alpha + \gamma = 2\beta$ ....(1) ; $\alpha + \beta + \gamma = \frac{-81}{18}$ ....(2)

$\alpha\beta\gamma = \frac{-60}{18}$ ....(3)

(1), (2) $\Rightarrow \beta = \frac{-3}{2}$ Put in (1), (3)

$\Rightarrow \quad \alpha + \gamma = -3 \quad \Rightarrow \quad \alpha\gamma = \frac{20}{9}$

$\therefore \quad x^2 - (-3)x + \frac{20}{9} = 0 \begin{smallmatrix}\alpha \\ \gamma\end{smallmatrix} \quad \Rightarrow \quad x = \frac{-3 \pm \sqrt{9 - 4\cdot 1\cdot \frac{20}{9}}}{2} = \frac{-5}{3}, \frac{-4}{3}$.

$\therefore$ roots are $\frac{-4}{3}, \frac{-3}{2}, \frac{-5}{3}$.

B-4. $\alpha^3 - 6\alpha^2 + 10\alpha - 3 = 0$.

Let $x = 2\alpha + 1$ new root $\alpha = \frac{x-1}{2} \Rightarrow \frac{(x-1)^3}{8} - \frac{6(x-1)^2}{4} + 5(x-1) - 3 = 0$

$(x^3 - 3x^2 + 3x - 1) - 12(x^2 - 2x + 1) + 40(x - 1) - 24 = 0 \Rightarrow x^3 - 15x^2 + 67x - 77 = 0$.

B-5. $2x^3 + x^2 - 7 = 0 \begin{smallmatrix}\alpha \\ \beta \\ \gamma\end{smallmatrix} \Rightarrow \alpha + \beta + \gamma = -1/2, \sum\alpha\beta = 0$, $\alpha\beta\gamma = 7/2$

$\sum\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\beta}{\gamma} + \frac{\gamma}{\beta} + \frac{\gamma}{\alpha} + \frac{\alpha}{\gamma} = \frac{1}{\beta}(\alpha + \gamma) + \frac{1}{\alpha}(\beta + \gamma) + \frac{1}{\gamma}(\alpha + \beta)$

$= \frac{1}{\beta}\left(-\frac{1}{2} - \beta\right) + \frac{1}{\alpha}\left(-\frac{1}{2} - \alpha\right) + \frac{1}{\gamma}\left(-\frac{1}{2} - \gamma\right) = -\frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right) - 1 - 1 - 1 = -\frac{1}{2}\left(\frac{\Sigma\alpha\beta}{\alpha\beta\gamma}\right) - 3 = -3$

B-6. Let roots be α, α and β

$\therefore \quad \alpha + \alpha + \beta = -\frac{20}{4} \quad \Rightarrow \quad 2\alpha + \beta = -5$ ....(1)

$\therefore \quad \alpha \cdot \alpha + \alpha\beta + \alpha\beta = -\frac{23}{4} \Rightarrow \quad \alpha^2 + 2\alpha\beta = -\frac{23}{4}$ ....(2)

and $\quad \alpha^2\beta = -\frac{6}{4} = -\frac{3}{2}$ ....(3)

from equation (1) put $\beta = -5 - 2\alpha$ in (2), we get $\alpha^2 + 2\alpha(-5 - 2\alpha) = -\frac{23}{4}$

$\Rightarrow \quad 12\alpha^2 + 40\alpha - 23 = 0 \quad \therefore \quad \alpha = 1/2, -\frac{23}{6}$

(i) If $\alpha = \frac{1}{2}$ then from (1), we get $\beta = -6$

(ii) If $\alpha = -\frac{23}{6}$ then from (1), we get $\beta = \frac{8}{3}$

Note : $\because$ $\alpha = \frac{1}{2}$ and $\beta = -6$ also satisfy (3) but $\alpha = -\frac{23}{6}$ and $\beta = \frac{8}{3}$ does not satisfy (3)

$\therefore$ required roots are $\frac{1}{2}, \frac{1}{2}, -6$

C-1. $2 + i\sqrt{3}$ and $2 - i\sqrt{3}$ are the roots of $x^2 + px + q = 0$

$\Rightarrow$ $-p = 4 \Rightarrow p = -4$ & $q = 7$.

C-2. $x^2 - 2cx + ab = 0$ has roots real and unequal i.e. $D_1 > 0 \Rightarrow 4c^2 - 4ab > 0 \Rightarrow c^2 - ab > 0$ ....(1)

Now, $x^2 - 2(a + b)x + (a^2 + b^2 + 2c^2) = 0$

$\Rightarrow$ $D_2 = 4(a + b)^2 - 4(a^2 + b^2 + 2c^2) = -8(c^2 - ab)$

by (1) $D_2 < 0$ roots will be imaginary.

C-3. $D = 0 \Rightarrow (k + 1)^2 - 8k = 0 \Rightarrow k^2 + 1 - 6k = 0 \Rightarrow k = \frac{6 \pm \sqrt{36-4}}{2} \Rightarrow k = 3 \pm 2\sqrt{2}$.

C-4. $D = 0 \Rightarrow 4(b^2 - ac)^2 - 4(a^2 - bc)(c^2 - ab) = 0 \Rightarrow b(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow$ Either $b = 0$ or $a^3 + b^3 + c^3 = 3abc$.

C-5. $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$ ...(1)

$\Rightarrow$ $(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$

$\Rightarrow$ $3x^2 - 2(a + b + c)x + ab + bc + ac = 0$ ...(2)

$D = 4(a + b + c)^2 - 12(ab + bc + ac) = 4[a^2 + b^2 + c^2 - ab - bc - ac] = 2[(a - b)^2 + (b - c)^2 + (c - a)^2]$

$\therefore$ $D \geq 0 \Rightarrow$ roots are always real But if $a = b = c$

Then $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0 \Rightarrow \frac{3}{x-a} = 0$

which has no real 'x'

$\Rightarrow$ this equation cannot have roots if $a = b = c$. $a = b = c$

C-6. $\frac{1}{(x+p)} + \frac{1}{(x+q)} = \frac{1}{r} \Rightarrow x^2 + x(p + q - 2r) + (pq - pr - qr) = 0 \begin{smallmatrix} \alpha \\ -\alpha \end{smallmatrix}$

$\therefore$ $\alpha + (-\alpha) = -(p + q - 2r) = 0 \Rightarrow p + q = 2r$

&Product of roots $= pq - r(p + q) = pq - r(p + q) = pq - \frac{(p+q)^2}{2} = -\frac{1}{2}(p^2 + q^2)$

C-7. **(i)** Roots are $-2 + i\beta, -2 - i\beta, \gamma$ (say) ; Sum of roots $(-2 + i\beta) + (-2 - i\beta) + \gamma = 0$; $\gamma = 4$.

Sum of products taken two at a time.

$4(-2 + i\beta) + 4(-2 - i\beta) + (4 + \beta^2) = 63$; $-16 + 4 + \beta^2 = 63$; $\beta^2 = 75$

$\beta = \pm 5\sqrt{3}$. Roots are $4, -2 \pm i5\sqrt{3}$.

(ii) Call roots as $\alpha, \frac{-1}{2} + i\beta, \frac{-1}{2} - i\beta$

$\alpha - 1 = \frac{-b}{2}$ ....(1)

$\alpha\left(\frac{-1}{2} + i\beta\right) + \alpha\left(\frac{-1}{2} + i\beta\right) + \frac{1}{4} + \beta^2 = \frac{3}{2}$ ....(2)

$\alpha\left(\frac{1}{4} + \beta^2\right) = \frac{-1}{2}$ ....(3)

(2) $\Rightarrow \frac{1}{4} + \beta^2 = \frac{3}{2} + \alpha$

Put in (3) $\alpha\left(\frac{3}{2} + \alpha\right) = \frac{-1}{2}$; $\alpha(2\alpha + 3) = -1$. $\Rightarrow$ $\alpha = -1, \frac{-1}{2}$.

If $\alpha = -1$, (3) $\Rightarrow b = 4$ $\therefore$ $\alpha = \frac{-1}{2} \Rightarrow b = 3$

Put in (1) b = 3 or 4

C-8. Given one root is –1 + i

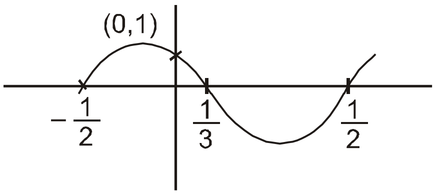
$\therefore$ 2^{nd} root will be –1 – i

$\therefore$ $x^2 + 2x + 2$ will be one factor of $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ and $x^2 + 2x - 1$ will be another factor

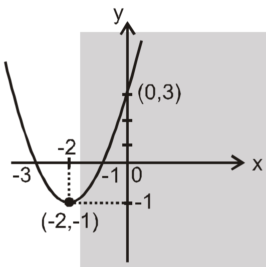
$\therefore$ The roots of given equation are $-1 \pm \sqrt{2}$ and $-1 \pm i$.

C-9. $y = (2x - 1)(6x^2 + x - 1) = (2x - 1)(2x + 1)(3x - 1)$. Hence roots are $x = -\frac{1}{2}, \frac{1}{3}, \frac{1}{2}$

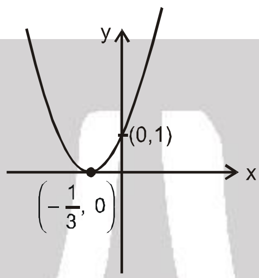
f(1) > 0



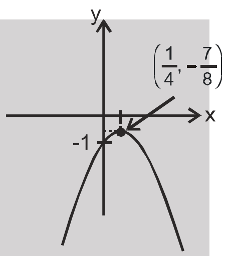
D-1. (i)



(ii)



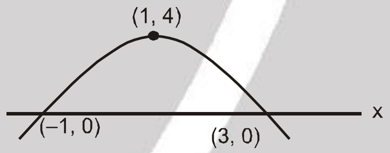
(iii)

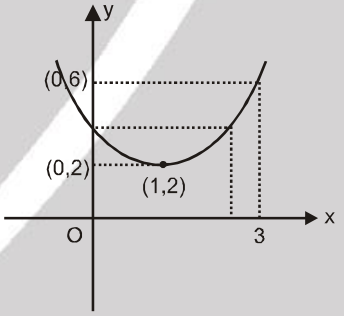


D-2. **(i)** $y = -x^2 + 2x + 3 = -(x^2 - 2x - 3) = -(x - 3)(x + 1)$

Here a < 0 and D > 0 $\Rightarrow$ Range is $(-\infty, 4]$

(ii) $f(x) = x^2 - 2x + 3 \quad \forall \quad x \in [0, \; 3]$





$y \in [2, \; 6] \; \forall \quad x \in [0, \; 3]$ **Ans.**

<u>Aliter :</u>

$f(x) = x^2 - 2x + 3 = (x - 1)^2 + 2$

Since $0 \le x \le 3 \Rightarrow -1 \le x - 1 \le 2 \Rightarrow 0 \le (x - 1)^2 \le 4$

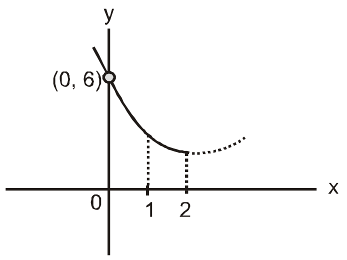
$\Rightarrow 2 \le (x - 1)^2 + 2 \le 6 \Rightarrow 2 \le f(x) \le 6$

$\therefore$ Range of f(x) is [2, 6].

(iii) $y = x^2 - 4x + 6$; $x \in (0, 1]$

Here a > 0 and D < 0

$f(0) = 6 \Rightarrow f(1) = 3 \Rightarrow$ Clearly for $x \in (0, 1] \Rightarrow y \in [3, 6)$



D-3. **(i)** $(y-1)x^2 - x + y - 1 = 0$

$\because$ $x \in R$

$\therefore$ $D \geq 0$

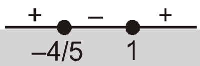
$\Rightarrow 1 - 4(y-1)^2 \geq 0 \Rightarrow (1 + 2y - 2)(1 - 2y + 2) \geq 0 \Rightarrow (2y-1)(2y-3) \leq 0 \Rightarrow \frac{1}{2} \leq y \leq \frac{3}{2}$

(ii) $y(x^2 - 2x - 9) = x^2 - 2x + 9 \quad \Rightarrow \quad (y-1)x^2 - 2(y-1)x - (y+1)9 = 0$

If $y = 1 \Rightarrow -(2)\,9 = 0$ contradiction.

$\therefore$ $y \neq 1$ $D \geq 0$ $\Rightarrow$ $(5y+4)(y-1) \geq 0$

$y \in \left(-\infty, \frac{-4}{5}\right] \cup (1, \infty)$



D-4. We can see for $x^2 - 8x + 17$

$D = 64 - 4(17) < 0$

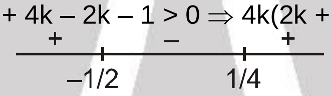
$\therefore$ $x^2 - 8x + 17$ is always +ve $\Rightarrow$ If $f(x) < 0$

$\therefore$ $kx^2 + 2(k+1)x + (9k+4) < 0 \Rightarrow k < 0$ ......(1)

& $4(k+1)^2 - 4k(9k+4) < 0 \Rightarrow k^2 + 1 + 2k - 9k^2 - 4k < 0 \Rightarrow -8k^2 - 2k + 1 < 0$

$8k^2 + 2k - 1 > 0 \Rightarrow 8k^2 + 4k - 2k - 1 > 0 \Rightarrow 4k(2k+1) - 1(2k+1) > 0$

$(2k+1)(4k-1) > 0$



.......(2)

combining (1) & (2) we get $k \in \left(-\infty, -\frac{1}{2}\right)$ (1) (2) $k \in \left(-\infty, -\frac{1}{2}\right)$

D-5. **(i)** $x^2 + (a-b)x + (1-a-b) = 0$

$\because$ $D > 0$

$\Rightarrow$ $(a-b)^2 - 4 \times 1 \times (1-a-b) > 0$

$\Rightarrow$ $a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

$\because$ $b^2 + 2b(2-a) + (a^2 + 4a - 4) > 0$

$\because$ $4(2-a)^2 - 4 \times 1 \times (a^2 + 4a - 4) < 0$

$4 + a^2 - 4a - a^2 - 4a + 4 < 0$

$\Rightarrow$ $8a - 8 > 0$ $\Rightarrow$ $a > 1$

(ii) $(a-b)^2 - 4 \cdot 1 \cdot (1-a-b) \leq 0$

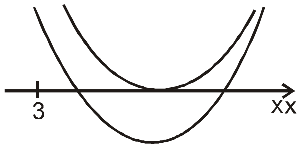
$\Rightarrow$ $b^2 + (4-2a)b + (a^2 + 4a - 4) \leq 0, \forall\, b \in R$

as cofficient of $b^2 = 1$, positive it in not possible.

$\therefore$ $a \in \phi$.

E-1. For both roots to exceed 3

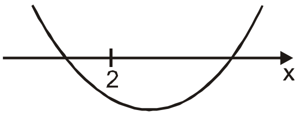
(i) $D \geq 0 \Rightarrow 36a^2 - 8 + 8a - 36a^2 \geq 0 \Rightarrow a \geq 1$



(ii) $f(3) > 0 \Rightarrow 9 - 18a + 2 - 2a + 9a^2 > 0 \Rightarrow 9a^2 - 20a + 11 > 0 \Rightarrow a \in (-\infty, 1) \cup \left(\frac{11}{9}, \infty\right)$

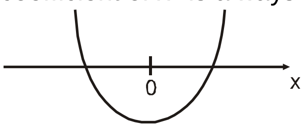
(iii) $\frac{-b}{2a} > 3 \Rightarrow 3a > 3 \Rightarrow a > 1$ $\therefore$ (i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow a > \frac{11}{9}$.

E-2. Here for one root to exceed 2 and other to be smaller than 2, f(2) < 0



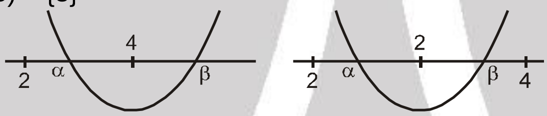
$\Rightarrow$ $4 - 2k - 2 + k^2 + k - 8 < 0$
$\Rightarrow$ $k^2 - k - 6 < 0$
$\Rightarrow$ $-2 < k < 3$.

E-3. Here coefficient of x^2 is always positive

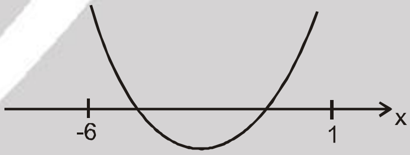


$\therefore$ $f(0) < 0$
$\Rightarrow$ $(a^2 + 4)(a - 2)(a + 2) < 0$
$\Rightarrow$ $a \in (-2, 2)$

E-4. (i) $D > 0$
$4a^2 - 4(a^2 - 1) > 0$
$4 > 0 \quad \forall\ x \in R$
(ii) $f(2)\ f(4) < 0$
$(4 - 4a + a^2 - 1)(16 - 8a + a^2 - 1) < 0$
$(a - 3)^2 (a - 1)(a - 5) < 0$
$a \in (1, 5) - \{3\}$



E-5. $x^2 + 2(k - 3)x + 9 = 0$ ......(i)
Roots α, β of equation (i) are distinct & lies between –6 and 1
$D > 0 \Rightarrow 4(K - 3)^2 - 36 > 0 \Rightarrow k(k - 6) > 0$
$\Rightarrow$ $k \in (-\infty, 0) \cup (6, \infty)$ ......(ii)
$f(1) > 0 \Rightarrow 1 + 2(k - 3) + 9 > 0$
$\Rightarrow$ $2k + 4 > 0$
$\Rightarrow$ $k \in (-2, \infty)$ ......(iii)
$f(-6) > 0 \Rightarrow 36 - 12(k - 3) + 9 > 0$
$\Rightarrow$ $4k - 27 < 0 \Rightarrow k \in \left(-\infty, \frac{27}{4}\right)$ ......(iv)



$-6 < -\frac{b}{2a} < 1 \Rightarrow -6 < \frac{-2(K-3)}{2} < 1$
$\Rightarrow -1 < k - 3 < 6 \qquad \Rightarrow 2 < k < 9$ ......(v)
(ii) $\cap$ (iii) $\cap$ (iv) $\cap$ (v) $\Rightarrow$ $k \in \left(6, \frac{27}{4}\right)$.

F-1. If α is one of the root of $a_1x^2 + b_1x + c_1 = 0$. Then $\frac{1}{\alpha}$ will be a root of $ax^2 + bx + c = 0$
$\Rightarrow c\alpha^2 + b\alpha + a = 0$ & $a_1\alpha^2 + b_1\alpha + c_1 = 0$ have one common root.
$\therefore$ applying the condition for one common root we get $(aa_1 - cc_1)^2 = (bc_1 - ab_1)(b_1c - a_1b)$

F-2. Given equation are

$x^2 - 11x + a = 0$ ....(i)

$x^2 - 14x + 2a = 0$ ....(ii)

Multiplying equation (i) by 2 and then subtracting, we get $x^2 - 8x = 0 \Rightarrow x = 0, 8$

If $x = 0$, $a = 0$

If $x = 8$, $a = 24$

F-3. $ax^2 + bx + c = 0 \Rightarrow bx^2 + cx + a = 0$ have a common root, say α

$\therefore \quad a\alpha^2 + b\alpha + c = 0 \Rightarrow \quad b\alpha^2 + c\alpha + a = 0 \Rightarrow \dfrac{\alpha^2}{ab - c} = \dfrac{\alpha}{bc - a^2} = \dfrac{1}{ac - b^2}$

$\alpha^2 = \dfrac{ab - c^2}{ac - b^2}, \quad \alpha = \dfrac{bc - a^2}{ac - b^2} \Rightarrow \left(\dfrac{ab - c^2}{ac - b^2}\right) = \left(\dfrac{bc - a^2}{ac - b^2}\right)^2 \Rightarrow \quad (ab - c^2)(ac - b^2) = (bc - a^2)^2$

$\Rightarrow a^3 + b^3 + c^3 = 3abc \qquad [\because a \neq 0] \qquad \Rightarrow \dfrac{a^3 + b^3 + c^3}{abc} = 3$ Ans.

<u>Aliter :</u>

By observation, x = 1 is the common root

$\therefore$ a + b + c = 0 $\therefore$ $a^3 + b^2 + c^3 = 3abc$ or = 3.

F-4. Let α is the common root hence $\alpha^2 + p\alpha + q = 0 \Rightarrow \alpha^2 + q\alpha + p = 0$

$\dfrac{\alpha^2}{p^2 - q^2} = \dfrac{\alpha}{q - p} = \dfrac{1}{q - p} \Rightarrow \alpha^2 = -(p + q), \quad \alpha = 1 \quad \Rightarrow -(p + q) = 1 \Rightarrow p + q + 1 = 0$

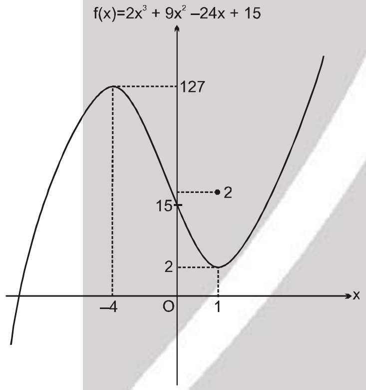
Let other roots be β and δ then $\alpha + \beta = -p$, $\alpha\beta = q \Rightarrow \alpha + \delta = -q$, $\alpha\delta = p$

$\beta - \delta = q - p, \quad \dfrac{\beta}{\delta} = \dfrac{q}{p} \Rightarrow \dfrac{\beta - \delta}{\delta} = \dfrac{q - p}{p} \Rightarrow \dfrac{q - p}{\delta} = \dfrac{q - p}{p} \Rightarrow \quad \delta = p \Rightarrow \beta = q$

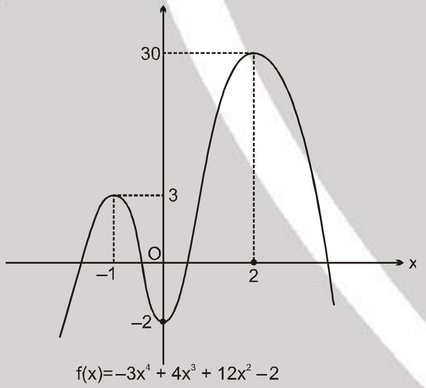
Equation having β, δ as roots

$x^2 - (\beta + \delta)x + \beta\delta = 0 \quad \Rightarrow x^2 - (p + q)x + pq = 0 \quad \Rightarrow \quad x^2 + x + pq = 0\ [\because p + q = -1]$

F-5. (i)



(ii)



F-6. $f(x) = x^3 - 3x^2 + 2$

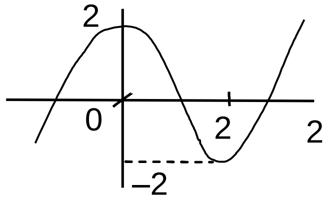
$f'(x) = 3x^2 - 6x = 3x(x - 2) = 0$

$f(0) = 2$

$f(2) = 8 - 12 + 2 = -2$

(i) $k \in [-2, 2]$

(ii) $k \in (-\infty, -2)\ (2, \infty)$



PART - II

A-1. x = 1 is root. Let other root = α

$\therefore$ Product of the roots = (1) (α) = $\frac{a-b}{b-c}$ $\Rightarrow$ roots are 1, $\frac{a-b}{b-c}$

A-2. $\alpha + \beta = -p \Rightarrow \alpha\beta = q \Rightarrow \gamma + \delta = -p \Rightarrow \gamma\delta = -r$

$(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta = \alpha^2 + p\alpha - r = \alpha(\alpha + p) - r = -\alpha\beta - r = -q - r = -(q + r)$

A-3. $(\alpha - \beta) = 4 \Rightarrow (\alpha - \beta)^2 = 16 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 16$

$\Rightarrow 9 - 4\alpha\beta = 16 \Rightarrow \alpha\beta = -\frac{7}{4} \Rightarrow$ equation is $x^2 - 3x - \frac{7}{4} = 0$

A-4. $3x^2 + px + 3 = 0 \genfrac{}{}{0pt}{}{\alpha^2}{\alpha}$ $\because$ $\alpha + \alpha^2 = -\frac{p}{3}$ .... (i)

$\alpha^3 = 1, \Rightarrow \alpha = 1, \omega, \omega^2$ $\because$ $\alpha \neq 1$

$\therefore$ $\alpha = \omega$ or $\alpha = \omega^2$ put is (i) $\therefore$ p = 3

A-5. S_1 : $x^2 - bx + c = 0 \genfrac{}{}{0pt}{}{\alpha}{\beta}$

$\because$ $|\alpha - \beta| = 1 \Rightarrow (\alpha - \beta)^2 = 1 \Rightarrow b^2 - 4c = 1.$

S_2 : $\because$ $\alpha + \beta = 1$ and $\alpha\beta = 3$

$\because$ $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 = (1 - 6)^2 - 2(9) = 25 - 18 = 7$

S_3 : $\because$ $\Sigma\alpha = 7 \Rightarrow \Sigma\alpha\beta = 16 \Rightarrow \alpha\beta\gamma = 12$

$\therefore$ $\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta) = 49 - 32$

$\therefore$ $\alpha^2 + \beta^2 + \gamma^2 = 17$

B-1. Let the roots be $\alpha, \beta, -\beta$ then $\alpha + \beta - \beta = p$

$\Rightarrow \alpha = p$ ...(1) and $\alpha\beta - \alpha\beta - \beta^2 = q \Rightarrow \beta^2 = -q$ ...(2)

also $-\alpha\beta^2 = r \Rightarrow pq = r$ [using (1)].

B-2. $x^3 - x - 1 = 0$ (roots α, β, γ) then $\alpha^3 - \alpha - 1 = 0$ ....(1)

Let $\frac{1+\alpha}{1-\alpha} = y \Rightarrow \alpha = \frac{y-1}{y+1}$ from equation (1) $\left(\frac{y-1}{y+1}\right)^3 - \left(\frac{y-1}{y+1}\right) - 1 = 0$ $y^3 + 7y^2 - y + 1 = 0$ (roots $\frac{1+\alpha}{1-\alpha}$, $\frac{1+\beta}{1-\beta}$, $\frac{1+\gamma}{1-\gamma}$)

then $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma} = -7$ **Ans.**

B-3. Clearly $(x - a)(x - b)(x - c) = -(x - \alpha)(x - \beta)(x - \gamma)$

$\therefore$ if α, β, γ are the roots of given equation

then $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ will have roots a, b, c.

B-4_. $\alpha + \beta + \gamma = 0 \Rightarrow \frac{\alpha^3 + \beta^3 + \gamma^3}{\alpha^2 + \beta^2 + \gamma^2} = \frac{3\alpha\beta\gamma}{-2(\alpha\beta + \beta\gamma + \gamma\alpha)} = \frac{3b}{2a}$

B-5_. Let roots are $\alpha, -\alpha, \beta, \gamma$ then $\beta + \gamma = 2$ and $-\alpha^2(\beta + \gamma) = -8$

$\Rightarrow \alpha^2 = 4 \Rightarrow \alpha \pm 2$

$\Rightarrow 2^4 - 2(2^3) + a(2)^2 + 8(2) + b = 0$

$\Rightarrow 4a + b = -16$

C-1. $\alpha + \beta = \sqrt{3}$

$\sqrt{3} + 2 + \beta = \sqrt{3} \quad \Rightarrow \beta = -2$

C-2. $D_1 = b^2 - 4.2.c > 0 \Rightarrow b^2 - 8c > 0$

$D_2 = (b - 4c)^2\ 4.2c.\ (2c - b + 1) = b^2 + 16c^2 - 8bc - 16c^2 + 8bc - 8c = b^2 - 8c > 0$

C-3. $\alpha + \alpha^2 = -\ell, \alpha^3 = m$

$\alpha^6 + \alpha^3 + 3\alpha^2 (\alpha + \alpha^2) = -\ell^3$

$\Rightarrow \quad m^2 + m + 3m(-\ell) + \ell^3 = 0 \quad \Rightarrow \quad m^2 + m(1 - 3\ell) + \ell^3 = 0$

$\Rightarrow \quad (1-3\ell)^2 - 4\ell^3 \geq 0$ {because $m \in R$}

$\Rightarrow \quad 4\ell^3 - 9\ell^2 + 6\ell - 1 \leq 0 \quad \Rightarrow \quad (\ell - 1)^2 (4\ell - 1) \leq 0 \quad \Rightarrow \quad \ell \in (-\infty, \frac{1}{4}] \cup \{1\}$

C-4. $D = b^2 - 4ac = 20d^2 \quad \Rightarrow \quad \sqrt{D} = 2\sqrt{5}d$ So roots are irrational.

C-5. $D = b^2 - 4ac = b^2 - 4a(-4a - 2b) = b^2 + 16a^2 + 8ab$

Since $ab > 0 \Rightarrow D > 0$. So equation has real roots.

C-6. For integral roots, D of equation should be perfect sq.

$\because \quad D = 4(1+n)$

By observation, for $n \in N$, D should be perfect sq. of even integer.

So $D = 4(1+n) = 6^2, 8^2, 10^2, 12^2, 14^2, 16^2, 18^2, 20^2$. No. of values of n = 8.

D-1. $x^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$\because \quad \alpha + \beta = -b$

$\Rightarrow \quad \alpha\beta = c$

$\because$ Sum is +ve and product is – ve.

$\therefore \quad \alpha < 0 < \beta < |\alpha|$

D-2. $a > 0$ & $c < 0$ is satisfied by (B) only [$\because$ f(0) = 0 & a > 0] Further in (B)

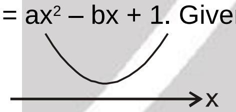
$-\frac{b}{2a} > 0 \quad \Rightarrow \quad b < 0 \quad [\because a > 0].$

D-3. For $y = ax^2 + bx + c$ to have the sign always same of 'a' $b^2 - 4ac < 0 \Rightarrow 4ac > b^2$

D-4. Here for D < 0, entire graph will be above x-axis ($\because a > 0$)

$\Rightarrow \quad (k-1)^2 - 36 < 0 \Rightarrow \quad (k-7)(k+5) < 0 \quad \Rightarrow \quad -5 < k < 7$

D-5. Let $f(x) = ax^2 - bx + 1$. Given D < 0 & f (0) = 1 > 0



$\therefore$ possible graph is as shown

i.e. $f(x) > 0 \ \ x \in R$ or $f(-1) > 0 \Rightarrow f(-1) = a + b + 1 > 0$

D-6. $x^2 + ax + b = 0 \quad \Rightarrow \quad a + b = -a \quad \Rightarrow \quad 2a + b = 0$ and $ab = b$

$ab - b = 0 \quad \Rightarrow \quad b(a-1) = 0 \quad \Rightarrow$ Either $b = 0$ or $a = 1$

But $b \neq 0$ (given)

$\therefore \quad a = 1$

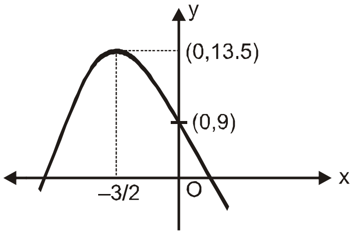
$\therefore \quad b = -2$

$\therefore \quad f(x) = x^2 + x - 2$

Least value occurs at $x = -\frac{1}{2}$

Least value $= \frac{1}{4} - \frac{1}{2} - 2 = -\frac{9}{4}$

D-7.



$y = -2x^2 - 6x + 9$

$\therefore \quad \frac{-b}{2a} = \frac{6}{2(-2)} = -\frac{3}{2} = -1.5$ & D = 36 – 4(–2) (9) = 36 + 72 = 108

$\therefore \quad -\frac{D}{4a} = -\frac{108}{4(-2)} = +\frac{108}{8} = 13.5$

$\Rightarrow \; y \in (-\infty, 13.5]$

D-8. min. f(x) > max. g(x)

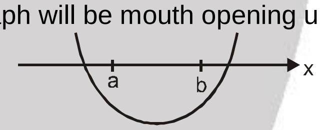
$\Rightarrow \quad -b^2 + 2c^2 > b^2 + c^2$

$\Rightarrow \quad c^2 > 2b^2$

$\Rightarrow \quad |c| > |b|\sqrt{2}$

E-1. $(x - a)(x - b) - 1 = 0$. Let $f(x) = (x - a)(x - b) - 1 \Rightarrow f(a) = -1 \Rightarrow f(b) = -1$

$\because$ the graph will be mouth opening upwards.



$\therefore$ (D) will be correct

E-2. $x^2 - 2px + (8p - 15) = 0$

f(1) < 0 and f(2) < 0

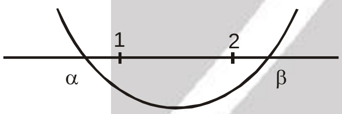
$\Rightarrow f(1) = 1 - 2p + 8p - 15 < 0$

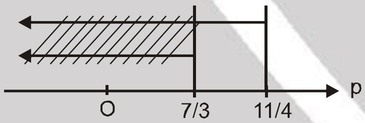
$\Rightarrow p < 7/3$

and $f(2) = 4 - 4p + 8p - 15 < 0$

$\Rightarrow 4p - 11 < 0 \quad \Rightarrow p < \frac{11}{4}$

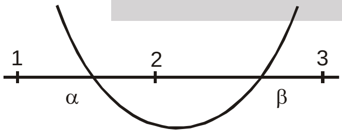
Hence $p \in (-\infty, \; 7/3)$ Ans.

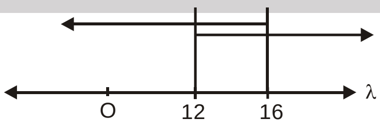




E-3. $4x^2 - 16x + \lambda = 0$

f(1) > 0 and f(2) < 0 and f(3) > 0





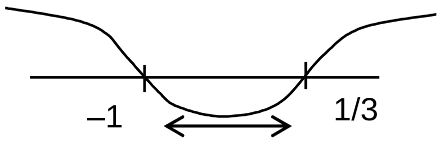
$f(1) = 4 - 16 + \lambda > 0$	$\Rightarrow$	$\lambda > 12$	...(i)
$f(2) = 16 - 32 + \lambda < 0$	$\Rightarrow$	$\lambda < 16$	...(ii)
$f(3) = 36 - 48 + \lambda > 0$	$\Rightarrow$	$\lambda > 12$	...(iii)

by (i) $\cap$ (ii) $\cap$ (iii)

$12 < \lambda < 16$. So $\lambda = 13, 14, 15$ has 3 integral solutions.

E-4. $D \geq 0$

$(k-1)^2 - 4k^2 \geq 0 \Rightarrow (k + 1)(3k - 1) \leq 0$



Case- I Exactly one in (1, 2)

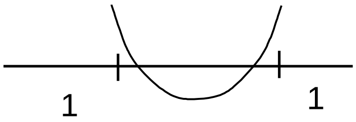
$f(1)\, f(2) < 0 \Rightarrow (1- k+1+1)(4-2k+2+k^2)<0$

$\Rightarrow (3 - k)(k^2 - 2k + 6) < 0$

$\Rightarrow 3 - k < 0 \Rightarrow k > 3$

if one roots is – 1 then k = 3

$-1 \times k = 9 \Rightarrow k = -9 \Rightarrow k \neq 3$



if one root is 2 then $k^2 - 2k + 6 = 0$ not possible

$\Rightarrow k \in \phi$

Case-II If both roots lie in (1, 2)

$f(1) > 0$ & $f(2) > 0$

$3 - k > 0 \Rightarrow k < 3$ & $k^2 - 2k + 6 > 0 \Rightarrow k \in \phi$

F-1. $x^2(6k + 2) + rx + (3k - 1) = 0 \quad \Rightarrow \quad x^2(12k + 4) + px + 6k - 2 = 0$

For both roots common, $\dfrac{6k+2}{12k+4} = \dfrac{r}{p} = \dfrac{3k-1}{2(3k-1)}$

$\Rightarrow \dfrac{r}{p} = \dfrac{1}{2}$

$\Rightarrow 2r - p = 0$ Ans.

F-2. $3x^2 - 17x + 10 = 0 \quad \Rightarrow \quad x = \dfrac{2}{3}$ or 5

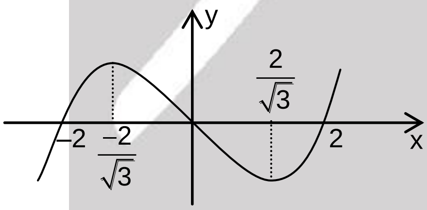
If $x = 5$ is comman $\Rightarrow \lambda = 0$

If $x = \dfrac{2}{3}$ is comman $\Rightarrow \lambda = \dfrac{26}{9}$; Sum = $\dfrac{26}{9}$

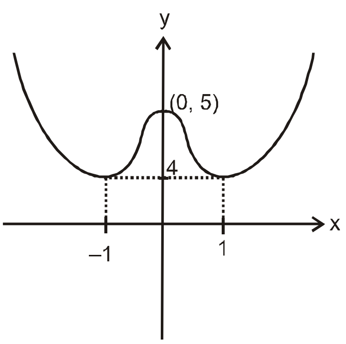
F-3. $D_1 = 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$ img. root ; $D_2 = 4p^2q^2 - 4p^2q^2 = 0$ equal, real roots

So no common roots.

F-4. (C)



F-5. (D)



PART - III

1. (A) $x^2 - 8x + k = 0 \begin{smallmatrix} \nearrow \alpha \\ \searrow \alpha + 4 = \beta \end{smallmatrix}$ $\quad \because \quad (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow \quad 16 = 64 - 4k \Rightarrow 4k = 48 \Rightarrow \quad k = 12$

(B) $x^2 + 2x - 4 = 0 \quad \Rightarrow \frac{1}{x^2} + \frac{2}{x} - 4 = 0$ has roots $\frac{1}{\alpha}, \frac{1}{\beta} \quad \Rightarrow \quad -4x^2 + 2x + 1 = 0$

$4x^2 - 2x - 1 = 0 \Rightarrow x^2 - \frac{1}{2}x - \frac{1}{4} = 0 \Rightarrow \quad q + r = \frac{-1}{2} - \frac{1}{4} = \frac{-3}{4} \Rightarrow 4 = \frac{-3}{q+r}$

(C) $\alpha + \beta = 0,\ \alpha\beta = \frac{c}{a} \Rightarrow \quad \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = 0.$

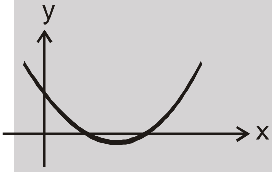
(D) $x^2 - kx + 36 = 0 \begin{smallmatrix} \nearrow \alpha \\ \searrow \beta \end{smallmatrix} \Rightarrow \alpha + \beta = k,\ \alpha\beta = 36 \Rightarrow 36 = (1)(36) = (2)(18) = (3)(12) = (4)(9) = (6)(6)$

or $36 = (-1)(-36) = (-2)(-18) = (-3)(-12) = (-4)(-9) = (-6)(-6)$ i.e. 10 values of k are possible.

2. $f(x) = ax^2 + bx + c\ (a \neq 0) \qquad a + b = -b/a$

$D = b^2 - 4ac \qquad \alpha b = c/a$

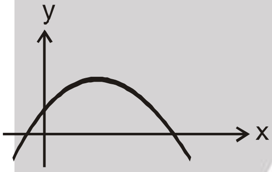
(P)



from graph

$D > 0\ \frac{-b}{a} > 0 \quad \Rightarrow \quad b < 0 \quad ; \quad a > 0 \quad \frac{c}{a} > 0 \quad \Rightarrow \quad c > 0$

(q)

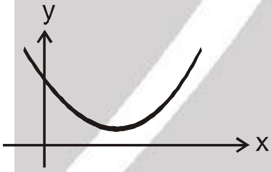


$D > 0$

$a < 0 \qquad abc < 0$

$f(0) > 0 \quad \Rightarrow \quad c > 0,\ \frac{abc}{D} < 0 \quad \Rightarrow \frac{-b}{2a} > 0 \quad \Rightarrow b > 0$

(r)

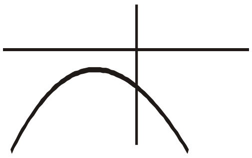


$D < 0$

$f(0) > 0 \quad \Rightarrow \quad C > 0 \quad abc < 0$

$\Rightarrow \quad a > 0 \quad \frac{abc}{D} > 0 \quad \Rightarrow \frac{-b}{2a} > 0 \quad \Rightarrow b < 0$ (A, D)

(s)



$D < 0$

$a < 0$

$f(0) < 0 \Rightarrow \quad C < 0 \quad \Rightarrow \frac{-b}{2a} < 0 \quad \Rightarrow \quad b < 0$

$abc < 0 \Rightarrow \quad \frac{abc}{D} > 0$ (A, D)

3. **(A) q, s, t (B) p, t (C) r (D) q, s.**

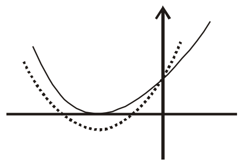
Exercise # 2

PART - I

1. $a > 0, b > 0$ and $c > 0 \quad \Rightarrow \quad ax^2 + bx + c = 0 \;\begin{matrix}\alpha \\ \beta\end{matrix}$

$\alpha + \beta = -b/a = -$ ve, $\alpha\beta = \dfrac{c}{a} = +$ ve

–ve real part



2. $x^2 + 2ax + b = 0 \;\begin{matrix}\alpha \\ \beta\end{matrix} \quad \Rightarrow \quad 0 < |\alpha - \beta| \le 2m \quad \Rightarrow \quad 0 < \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} \le 2m$

$0 < 4a^2 - 4b \le 4m^2 \quad \Rightarrow \quad a^2 - m^2 \le b < a^2 \quad \Rightarrow \quad b \in [a^2 - m^2, a^2)$

3. Sum of roots < 1

$\Rightarrow \quad \lambda^2 - 5\lambda + 5 < 1 \quad \Rightarrow \quad (\lambda - 1)(\lambda - 4) < 0 \quad \Rightarrow \quad 1 < \lambda < 4$ ...(1)

$\Rightarrow$ Product of roots < 1

$\Rightarrow \quad 2\lambda^2 - 3\lambda - 5 < 0 \quad \Rightarrow \quad (2\lambda - 5)(\lambda + 1) < 0 \quad \Rightarrow \quad -1 < \lambda < \dfrac{5}{2}$ ...(2)

(1) & (2) $\quad \Rightarrow \quad 1 < \lambda < \dfrac{5}{2}$.

4. Dis. of $x^2 + px + 3q$ is $p^2 - 12q \equiv D_1$

Dis. of $-x^2 + rx + q$ is $r^2 + 4q \equiv D_2$

Dis. of $-x^2 + sx - 2q$ is $s^2 - 8q \equiv D_3$

Case 1 : If $q < 0$, then $D_1 > 0$, $D_3 > 0$ and D_2 may or may not be positive

Case 2 : If $q > 0$, then $D_2 > 0$ and D_1, D_3 may or may not be positive

Case 3 : If $q = 0$, then $D_1 \ge 0$, $D_2 \ge 0$ and $D_3 \ge 0$

from **Case 1, Case 2** and **Case 3** we can say that the given equation has atleast two real roots.

5. We, know that $a + b > c$, $b + c > a$ and $c + a > b \quad \Rightarrow \quad c - a < b$, $a - b < c$, $b - c < a$

squaring on both sides and adding $(c - a)^2 + (a - b)^2 + (b - c)^2 < a^2 + b^2 + c^2$

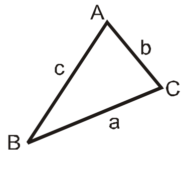
$a^2 + b^2 + c^2 - 2(ab + bc + ca) < 0 \quad \Rightarrow \quad (a + b + c)^2 - 4(ab + bc + ca) < 0$

$\Rightarrow \quad \dfrac{(a+b+c)^2}{ab+bc+ca} < 4$ ....(i)

Now roots of equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then $D \ge 0$

$\Rightarrow \quad 4(a + b + c)^2 - 4.\, 3\lambda(ab + bc + ca) \ge 0 \quad \Rightarrow \quad \dfrac{(a+b+c)^2}{ab+bc+ca} \ge 3\lambda$

So $\quad 3\lambda \le \dfrac{(a+b+c)^2}{ab+bc+ca} < 4 \quad \Rightarrow \quad \lambda < \dfrac{4}{3}$



6. Let biqhadratic is $ax^4 + bx^3 + cx^2 + dx + e = 0$
$\Rightarrow$ $a + b + c + d + e = 0$ as $a, b, c, d, e \in \{-9, -5, 3, 4, 7\}$
Hence $x = 1$ is a root. So real root will be atleast two.
$ax^4 + bx^3 + cx^2 + dx + e = 0 \Rightarrow a + b + c + d + e = 0$ $a, b, c, d, e \in \{-9, -5, 3, 4, 7\}$

7. $x^2 + px + q = 0 \Rightarrow D_1 = p^2 - 4q$ .....(1)
$x^2 + rx + s = 0 \Rightarrow D_2 = r^2 - 4s$ .....(2)
$D_1 + D_2 = p^2 + r^2 - 4(q + s)$ $[\because pr = 2(q + s)] = (p - r)^2 > 0$
Since $D_1 + D_2$ is +ve, so atleast
one of the equation has real roots.

8. $\pi^x = -2x^2 + 6x - 9 \Rightarrow D = 36 - 4(-2)(-9) = 36 - 72 < 0$ & $a < 0$
So quadratic expression $-2x^2 + 6x - 9$ is always negative whereas π^x is always +ve
$\therefore$ Equation will not hold for any x.
$\therefore x \in \phi$ So $\pi^x = -2x^2 + 6x - 9$ has no solution.

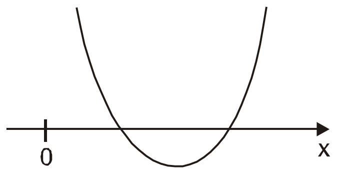
9. $(\lambda + 2)(\lambda - 1)x^2 + (\lambda + 2)x - 1 < 0 \ \forall x \in R \Rightarrow (\lambda + 2)(\lambda - 1) < 0$
$\Rightarrow -2 < \lambda < 1$ ...(1) $(a < 0)$
and $(\lambda + 2)^2 + 4(\lambda + 2)(\lambda - 1) < 0$ $(D < 0)$
$\Rightarrow (\lambda + 2)(\lambda + 2 + 4\lambda - 4) < 0 \Rightarrow (\lambda + 2)(5\lambda - 2) < 0$
$\Rightarrow -2 < \lambda < \frac{2}{5}$ ...(2)
(1) & (2) $\Rightarrow \lambda \in \left(-2, \frac{2}{5}\right)$ Also $\lambda = -2 \Rightarrow 0 < 1$ which is true
$\therefore$ Required interval is $\lambda \in \left[-2, \frac{2}{5}\right)$

10. $C_1 : b^2 - 4ac \ge 0$; $C_2 : a, -b, c$ are of same sign $ax^2 + bx + c = 0$ has real roots then $D \ge 0$ i.e. C_1 must be satisfied
(i) Let $a, -b, c > 0$ then $-\frac{b}{2a} > 0$
(ii) Let $a, -b, c < 0$ then $-\frac{b}{2a} > 0$
Hence, for roots to be + ve, C_2 must be satisfied. Thus both C_1, C_2 are satisfied

11. Let $y = \frac{x^2 - x + c}{x^2 + x + 2c}$; $x \in R$ and $y \in R$ $\Rightarrow (y - 1)x^2 + (y + 1)x + 2yc - c = 0$
$\because x \in R \Rightarrow D \ge 0 \Rightarrow (y + 1)^2 - 4c(y - 1)(2y - 1) \ge 0$
$\Rightarrow y^2 + 1 + 2y - 4c[2y^2 - 3y + 1] \ge 0 \Rightarrow (1 - 8c)y^2 + (2 + 12c)y + 1 - 4c \ge 0$ .... (1)
Now for all $y \in R$ (1) will be true if $1 - 8c > 0 \Rightarrow c < \frac{1}{8}$ and $D \le 0$
$\Rightarrow 4(1 + 6c)^2 - 4(1 - 8c)(1 - 4c) \le 0 \Rightarrow 1 + 36c^2 + 12c - 1 - 32c^2 + 12c \le 0$
$\Rightarrow 4c^2 + 24c \le 0 \Rightarrow -6 \le c \le 0$
But $c = -6$ and $c = 0$ will not satisfy given condition
$\therefore c \in (-6, 0)$

12. $(2 - x)(x + 1) = p \Rightarrow x^2 - x + (p - 2) = 0$ ...(1)
(1) has both roots distinct & positive
$\therefore$ (i) $D > 0$ (ii) $f(0) > 0$ (iii) $\frac{-b}{2a} > 0$
(i) $D > 0 \Rightarrow p < \frac{9}{4}$ (ii) $f(0) > 0 \Rightarrow p > 2$ (iii) $\frac{-b}{2a} = \frac{1}{2} > 0$ (always true)

$\therefore$ (i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow p \in \left(2, \dfrac{9}{4}\right)$.



13. $(a-1)(x^2+x+1)^2 - (a+1)(x^4+x^2+1) = 0$ ....(1)

$\because$ $x^4 + x^2 + 1 = (x^2 + x + 1)\ (x^2 - x + 1)$

$\therefore$ (1) becomes

$\Rightarrow$ $(x^2 + x + 1)\ [(x^2 + x + 1)\ (a - 1) - (a + 1)\ (x^2 - x + 1)] = 0$

$\Rightarrow$ $(x^2 + x + 1)\ (x^2 - ax + 1) = 0$

Here two roots are imaginary and for other two roots to be real D > 0

$\Rightarrow$ $a^2 - 4 > 0$ $\Rightarrow$ $a \in (-\infty, -2) \cup (2, \infty)$

14. $x^3 + 5x^2 + px + q = 0 \begin{cases}\alpha \\ \beta \\ x_1\end{cases}$ $\Rightarrow$ $\alpha + \beta + x_1 = -5, \alpha\beta + \beta x_1 + \alpha x_1 = p$ ...(1)

$x^3 + 7x^2 + px + r = 0 \begin{cases}\alpha \\ \beta \\ x_2\end{cases}$ $\Rightarrow$ $\alpha + \beta + x_2 = -7, \alpha\beta + \beta x_2 + \alpha x_2 = p$ ...(2)

Subtracting (2) from (1)

$\alpha\beta + \beta x_1 + \alpha x_1 = p$ $\Rightarrow$ $\dfrac{\alpha \quad \beta + \beta x_2 + \alpha x_2 = p}{\alpha \quad (x_1 - x_2) + \beta \quad (x_1 - x_2) = 0}$ $\Rightarrow$ $(x_1 - x_2)(\alpha - \beta) = 0\ [x_1 \neq x_2]$

$\therefore$ $\alpha + \beta = 0$ $\Rightarrow$ $x_1 = -5 \Rightarrow$ $x_2 = -7$

15. $\because$ $a^2 + b^2 + c^2 = 1$ $\Rightarrow$ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2\,(ab + bc + ca) \geq 0$

$\Rightarrow$ $1 + 2\,(ab + bc + ca) \geq 0$ $\Rightarrow$ $(ab + bc + ca) \geq -\dfrac{1}{2}$ ....(1)

$\because$ $a^2 + b^2 + c^2 - (ab + bc + ca) \geq 0$ $\Rightarrow$ $(ab + bc + ca) \leq 1$ ....(2)

$\therefore$ From (1) and (2) we can say that $(ab + bc + ca) \in \left[-\dfrac{1}{2}, 1\right]$

PART - II

1. $\left(x^2 + 3x + 2\right)\left(x^2 + 3x\right) = 120$

Let $x^2 + 3x = y \Rightarrow y^2 + 2y - 120 = 0$ $\Rightarrow$ $(y + 12)\ (y - 10) = 0$

$\Rightarrow$ $y = -12 \Rightarrow x^2 + 3x + 12 = 0$ $\Rightarrow$ $x \in \phi$

$y = 10 \Rightarrow x^2 + 3x - 10 = 0$ $\Rightarrow$ $(x + 5)\ (x - 2) = 0$

$\Rightarrow$ $x = \{-5, 2\}$

x = 2, – 5 are only two integer roots.

2. $\left(5 + 2\sqrt{6}\right)^{x^2-3} + \dfrac{1}{\left(5 + 2\sqrt{6}\right)^{x^2-3}} = 10$

$\Rightarrow$ $t + \dfrac{1}{t} = 10$

$\Rightarrow$ $t^2 - 10t + 1 = 0$

$t = \dfrac{10 \pm \sqrt{96}}{2} = 5 \pm 2\sqrt{6}$

$\Rightarrow$ $(5 + 2\sqrt{6})^{x^2-3} = (5 + 2\sqrt{6})$ or $\dfrac{1}{5 + 2\sqrt{6}}$

$\Rightarrow$ $x^2 - 3 = 1$ or $x^2 - 3 = -1$

$\Rightarrow$ $x = 2$ or -2 or $-\sqrt{2}$ or $\sqrt{2}$ Product 8

3. $x^2 + px + 1 = 0 \begin{matrix} a \\ b \end{matrix}$ $a + b = -p,\ ab = 1$; $x^2 + qx + 1 = 0 \begin{matrix} c \\ d \end{matrix}$ $c + d = -q,\ cd = 1$

$a + b = -p,\ ab = 1 \Rightarrow c + d = -q,\ cd = 1$
RHS $= (a - c)(b - c)(a + d)(b + d) = (ab - ac - bc + c^2)(ab + ad + bd + d^2)$
$= (1 - ac - bc + c^2)(1 + ad + bd + d^2)$
$= 1 + ad + bd + d^2 - ac - a^2cd - abcd - acd^2 - bc - abcd - b^2cd - bcd^2 + c^2 + adc^2 + bdc^2 + c^2d^2$
$= 1 + ad + bd + d^2 - ac - a^2 - 1 - ad - bc - 1 - b^2 - bd + c^2 + ac + bc + 1$ $[\because ab = cd = 1]$
$= c^2 + d^2 - a^2 - b^2 = (c + d)^2 - 2cd - (a + b)^2 + 2ab = q^2 - 2 - p^2 + 2 = q^2 - p^2 =$ LHS. **Proved.**

Aliter:
RHS $= (ab - c(a + b) + c^2)(ab + d(ab + d(a + b) + d^2) = (c^2 + pc + 1)(1 - pd + d^2)$ ...(1)
Since c & d are the roots of the equation $x^2 + qx + 1 = 0$
$\therefore\ c^2 + qc + 1 = 0 \Rightarrow c^2 + 1 = -qc$ & $d^2 + qd + 1 = 0 \Rightarrow d^2 + 1 = -qd$.
$\therefore$ (i) Becomes $= (pc - qc)(-pd - qd) = c(p - q)(-d)(p + q) = -cd(p^2 - q^2)$
$= cd(q^2 - p^2) = q^2 - p^2 =$ LHS. **Proved.**

4. $\because$ α, β are roots of $\lambda x^2 - (\lambda - 1)x + 5 = 0$

$\therefore$ $\alpha + \beta = \dfrac{\lambda - 1}{\lambda}$ and $\alpha\beta = \dfrac{5}{\lambda}$

$\because$ $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = 4 \Rightarrow \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = 4 \Rightarrow (\alpha + \beta)^2 = 6\alpha\beta$

$\Rightarrow \dfrac{(\lambda - 1)^2}{\lambda^2} = \dfrac{30}{\lambda} \Rightarrow \lambda^2 - 32\lambda + 1 = 0$ ....(1)

$\because$ λ_1, λ_2 are roots of (1) $\therefore$ $\lambda_1 + \lambda_2 = 32$ and $\lambda_1\lambda_2 = 1$

$\therefore$ $\dfrac{\lambda_1}{\lambda_2} + \dfrac{\lambda_2}{\lambda_1} = \dfrac{(\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2} = \dfrac{(32)^2 - 2}{1} = 1022 \Rightarrow \left(\dfrac{\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}}{14}\right) = 73$

5. $\alpha + 2\alpha = -\dfrac{\ell}{\ell - m} \Rightarrow \alpha = -\dfrac{\ell}{3(\ell - m)}$ Also $2\alpha^2 = \dfrac{1}{\ell - m} \Rightarrow \dfrac{2\ell^2}{9(\ell - m)^2} = \dfrac{1}{\ell - m}$

$\Rightarrow 2\ell^2 - 9\ell + 9m = 0 \Rightarrow \ell \in R \Rightarrow D \geq 0 \Rightarrow 81 - 72m \geq 0 \Rightarrow m \leq \dfrac{9}{8}$.

6. $\alpha.\beta = b$; $\gamma\delta = b - 2 \Rightarrow \alpha\beta\gamma\delta = b(b - 2) = 24$

$\therefore$ $bx^2 + ax + 1 = 0$ has roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta} \Rightarrow \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{-a}{b}$

$(b - 2)x^2 - ax + 1 = 0$ has root $\dfrac{1}{\gamma}, \dfrac{1}{\delta} \Rightarrow \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{a}{b - 2}$

$\dfrac{1}{\gamma} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{-a}{b} + \dfrac{a}{b - 2} = \dfrac{5}{6}$; $\dfrac{+2a}{b(b - 2)} = \dfrac{5}{6}$; $\dfrac{+2a}{24} = \dfrac{5}{6}$; $a = 10$.

7. $a^3 + b^3 + (-9)^3 = 3 \cdot a \cdot b(-9) \Rightarrow a + b - 9 = 0$ or $a = b = -9$. Which is rejected.
As $a > b > -9 \Rightarrow a + b - 9 = 0 \Rightarrow x = 1$ is a root

other root $= \dfrac{-9}{a}$. $\therefore$ $\alpha = \dfrac{-9}{a}$, $\beta = 1 \Rightarrow 4\beta - a\alpha = 4 - a\left(\dfrac{-9}{a}\right) = 4 + 9 = 13$.

8. Let $t^2 - 2t + 2 = k \Rightarrow \alpha^2 - 6k\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6k\alpha$
$a_{100} - 2a_{98} = \alpha^{100} - 2.\alpha^{98} - \beta^{100} + 2.\beta^{98} = \alpha^{98}(\alpha^2 - 2) - \beta^{98}(\beta^2 - 2) = 6k(\alpha^{99} - \beta^{99})$ $a_{100} - 2a_{98} = 6k.a_{99}$

$\dfrac{a_{100} - 2a_{98}}{a_{99}} = 6k = 6(t^2 - 2t + 2) = 6[(t - 1)^2 + 1]$ $\therefore$ min. value of $\dfrac{a_{100} - 2a_{98}}{a_{99}}$

9. $x^4 - Kx^3 + Kx^2 + Lx + M = 0 \begin{smallmatrix}\alpha \\ \beta \\ \gamma \\ \delta\end{smallmatrix} \Rightarrow \quad \sum \alpha = K, \ \sum \alpha\beta = K, \ \sum \alpha\beta\gamma = -L$

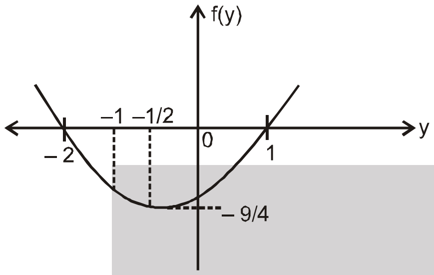
$\alpha\beta\gamma\delta = M \quad \Rightarrow \quad \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2\sum \alpha\beta$

$K^2 - 2K = (K - 1)^2 - 1 \quad \Rightarrow \quad (\alpha^2 + \beta^2 + \gamma^2 + \delta^2)_{min} = -1$

10. $y = \dfrac{2x}{1 + x^2} \quad \Rightarrow \quad x^2y - 2x + y = 0 \quad \forall \ x \in R$

$D \geq 0 \quad 4 - 4y^2 \geq 0 \quad \Rightarrow \quad y \in [-1, 1]$ Now $f(y) = y^2 + y - 2$

$\Rightarrow f(y) \in \left[-\dfrac{9}{4}, 0\right] \quad \Rightarrow \quad a = \dfrac{-9}{4}, b = 0 \Rightarrow b - 4a = 0 - 4\left(\dfrac{-9}{4}\right) = 9$. **Ans.**

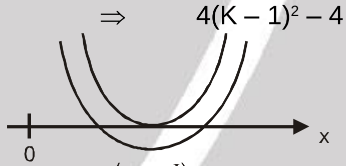


11. Let α, β, γ be the roots of $x^3 - Ax^2 + Bx - C = 0$ ...(1)
the roots of $x^3 + Px^2 + Qx - 19 = 0$ will be $(\alpha + 1), (\beta + 1), (\gamma + 1)$
$\therefore \quad (\alpha + 1)(\beta + 1)(\gamma + 1) = 19 \quad \Rightarrow \quad (\alpha\beta + \alpha + \beta + 1)(\gamma + 1) = 19$
$\Rightarrow \quad \alpha\beta\gamma + \alpha\gamma + \beta\gamma + \alpha\beta + \alpha + \beta + \gamma + 1 = 19 \quad \Rightarrow \quad C + B + A = 18$ [using (1)].

12. $\alpha + 2\alpha = 12x \quad \Rightarrow \quad \alpha = 4x \quad \Rightarrow \quad (\alpha)(2\alpha) = -f(x) - 64x$
$\Rightarrow \quad f(x) = -(32x^2 + 64x) \quad \Rightarrow \quad f(x) = -32(x^2 + 2x) \quad \Rightarrow \quad f(x) = -32((x + 1)^2 - 1)$
$\Rightarrow \quad f(x) \leq 32. \quad \Rightarrow$ Maximum value of f(x) is 32 f(x)

13. **Case-I :** Both the roots are positive $x^2 + 2(K - 1)x + (K + 5) = 0$

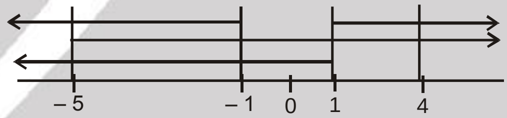
(i) $D \geq 0 \quad \Rightarrow \quad 4(K - 1)^2 - 4(K + 5) \geq 0 \Rightarrow \quad (K + 1)(K - 4) \geq 0$



(case-I)

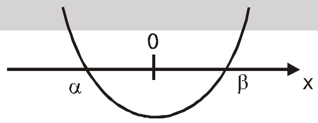
(ii) $f(0) > 0 \quad \Rightarrow \quad K + 5 > 0 \quad \Rightarrow \quad K > -5$

(iii) $-\dfrac{b}{2a} > 0 \quad \Rightarrow \quad \dfrac{2(1-k)}{2} > 0 \quad \Rightarrow \quad K < 1$



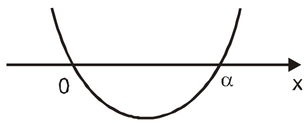
$K \in (-\infty, -1]$ ... (i)

Case-II : One root is +ve and other root is –ve $f(0) < 0 \Rightarrow k + 5 < 0 \Rightarrow \quad K < -5$ ... (ii)



(case-II)

Case-III : One root is zero and other is +ve $f(0) = 0$ & $\dfrac{-b}{2a} > 0 \quad \Rightarrow K = -5$ ... (iii)



(case-III)

Union of all the three cases give $K \in (-\infty, -1] = (-\infty, -b] \Rightarrow b = 1$. **Ans.**

14. **case- I :** Both roots are greater than 2.

or one root is 2 & other is greater than 2

$D \geq 0 \Rightarrow (a-3)^2 - 4a \geq 0 \Rightarrow a^2 - 10a + 9 \geq 0 \quad (a-1)(a-9) \geq 0$

$a \in (-\infty, 1] \cup [9, \infty)$ ... (i)

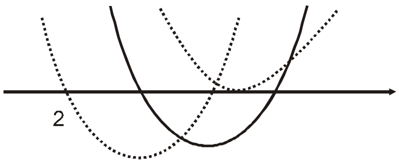
$\frac{-b}{2a} > 2 \Rightarrow \frac{a-b}{2} > 2 \Rightarrow a > 7$ .. (ii)

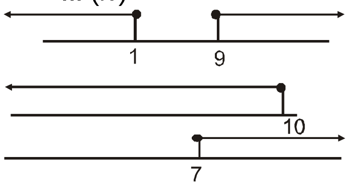
$f(2) \geq 0 \Rightarrow 4 - 2(a-3) + a \geq 0$

$-a + 10 \geq 0 \Rightarrow a \leq 10$ ... (iii)

(i) $\cap$ (ii) $\cap$ (iii) gives

$a \in [9, 10]$ ... (iv)





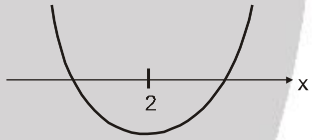
Case-II : One root is greater than 2

$f(2) < 0 \Rightarrow -a + 10 < 0$

$\Rightarrow a > 10 \Rightarrow a \in (10, \infty)$ ....(v)

(iv) $\cup$ (v) gives final answer as $a \in [9, \infty)$

$\Rightarrow$ Least value of 7a is 63.



15. $\begin{vmatrix} 3 & a \\ 2 & b \end{vmatrix} \begin{vmatrix} a & 1 \\ b & 1 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix}^2 \Rightarrow (3b - 2a)(a - b) = (3-2)^2$

$\Rightarrow 5ab - 3b^2 - 2a^2 = 1$

16. $x^3 - px^2 + qx = 0$ ...(1)

$x(x^2 - px + q) = 0; \quad x = 0, \quad x^2 - px + q = 0 \begin{matrix} \alpha \\ \alpha \end{matrix} \quad \therefore \; 0, \alpha, \alpha$ are the roots of equation (1)

$2\alpha = p \Rightarrow \alpha = p/2$ ...(2) & $\alpha^2 = q$ ...(3)

Since α is the root of the equation $x^2 - ax + b = 0$ also,

$\therefore \; \alpha^2 - a\alpha + b = 0$

$q - \frac{a\ p}{2} + b = 0$ [using (2) & (3)]

$\Rightarrow ap = 2(b+q) \qquad \Rightarrow 2 = \frac{ap}{q+b}$.

17. Given expression is $f(x, y) = x^3 - 3x^2y + \lambda xy^2 + \mu y^3$ .....(i)

since $(x - y)$ is a factor of (i)

$\therefore \; x^3 - 3x^3 + \lambda x^3 + \mu x^3 = 0 \Rightarrow \lambda + \mu - 2 = 0$ .....(ii)

$(y - 2x)$ is also a factor of (i)

$\therefore \; x^3 - 3x^2(2x) + \lambda x(4x^2) + \mu(8x^3) = 0$

$\Rightarrow 4\lambda + 8\mu - 5 = 0$ .....(iii)

Solving (ii) & (iii) we get $\lambda = \frac{11}{4}$ and $\mu = -\frac{3}{4}$

$\Rightarrow \frac{16\lambda}{11} + 4\mu = \frac{16}{11}\ \frac{11}{4} + 4\left(\frac{-3}{4}\right) = 4 - 3 = 1.$ **Ans.**

PART - III

1. p = 0 $\Rightarrow 2x^2 - 4x - 0 = 0$ two roots **(A) Correct**
p = 1 $\Rightarrow 0x^2 - (0)x + 0 = 0$ identity more than two roots **(B) Not answer**
p = 2 $\Rightarrow 0x^2 - (-2)x + (-2) = 0 \Rightarrow x = +1$ one root **(C) Correct**
p = 4 $\Rightarrow 6x^2 - 0x - 12 = 0$ two root **(D) Correct**

2. **(A)** $S = \alpha^2 + \beta^2 = a^2 - 2b$; $P = \alpha^2\beta^2 = b^2$
$\therefore$ equation is $x^2 - (a^2 - 2b)x + b^2 = 0$

(B) $S = \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{a}{b}$, $P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{b}$

$\therefore$ $x^2 + \frac{a}{b}x + \frac{1}{b} = 0$

$\Rightarrow$ $bx^2 + ax + 1 = 0$

(C) $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{a^2-2b}{b}$; $P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$x^2 - \frac{a^2-2b}{b}x + 1 = 0 \Rightarrow bx^2 - (a^2 - 2b)x + b = 0$

(D) $S = \alpha + \beta - 2 = -a - 2$; $P = (\alpha - 1)(\beta - 1)$
$= \alpha\beta - (\alpha + \beta) + 1 = b + a + 1$
$\therefore$ equation is $x^2 + (a + 2)x + (a + b + 1) = 0$.

3. $ax^2 + bx + c = 0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \beta\end{smallmatrix}$ $\Rightarrow \alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$ $\Rightarrow Ax^2 + Bx + C = 0 \begin{smallmatrix}\nearrow \alpha+\delta \\ \searrow \beta+\delta\end{smallmatrix}$

$(\alpha + \delta) + (\beta + \delta) = -\frac{B}{A}$, $(\alpha + \delta)(\beta + \delta) = \frac{C}{A}$ $\because$ $|(\alpha + \delta) - (\beta + \delta)| = |(\alpha - \beta)|$

$\Rightarrow \sqrt{\frac{B^2}{A^2} - \frac{4C}{A}} = \sqrt{\frac{b^2}{a^2} - \frac{4c}{a}} \Rightarrow \frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$ Hence proved

4. $4x^2 + 2x - 1 = 0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \beta\end{smallmatrix}$

$\Rightarrow$ $4\alpha^2 + 2\alpha - 1 = 0$ ....(1)
Let $\beta = 4\alpha^3 - 3\alpha$
with the help of equation (1)

$\beta = \alpha\,[4\alpha^2 - 3] = \alpha[1 - 2\alpha - 3] = -2\alpha^2 - 2\alpha = -2\,\frac{(1-2\alpha)}{4} - 2\alpha$ [using (1)]

$\beta = -\alpha - 1/2$
$\alpha + \beta = -1/2$ which is given.
hence second root is $4\alpha^3 - 3\alpha$.

5. $x^2 + 3x + 1 = (x - \alpha)(x - \beta)$. Put $x = 2$ $\Rightarrow 11 = (2 - \alpha)(2 - \beta)$ **option (B)**
$\alpha^2 + 3\alpha + 1 = 0$, $\beta^2 + 3\beta + 1 = 0$
$\alpha^2 = -(3\alpha + 1)$, $\beta^2 = -(3\beta + 1)$

$\frac{\alpha^2}{3\alpha+1} = -1$, $\frac{\beta^2}{3\alpha+1} = -1$ $\Rightarrow$ $\frac{\alpha^2}{3\alpha+1} + \frac{\beta^2}{3\alpha+1} = -2$ **option (C).**

$$\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{1+\alpha}\right)^2 = \frac{\alpha^2}{1+2\beta+\beta^2} + \frac{\beta^2}{1+2\alpha+\alpha^2} = \frac{-(3\alpha+1)}{-\beta} + \frac{-(3\beta+1)}{-\alpha} = \frac{\alpha\,(3\alpha+1)+\beta(3\beta+1)}{\beta\alpha}$$

$$= \frac{3(\alpha^2+\beta^2)+(\alpha+\beta)}{1} = \frac{3\left((\alpha+\beta)^2 - 2\alpha\beta\right)+(-3)}{1} = 3\,(7) - 3 = 18.$$

6. Split 32 into sum of two primes 32 = 2 + 30 = 3 + 29 = 5 + 27 = 7 + 25 = 11 + 21 = 13 + 19.
32 = 2 + 30 = 3 + 29 = 5 + 27 = 7 + 25 = 11 + 21 = 13 + 19.

7. $\alpha^2 - a(\alpha + 1) - b = 0$ ....(i)
$\beta^2 - a(\beta + 1) - b = 0$ ....(ii)
by (i) & (ii)

(A) $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} - \frac{2}{a+b} = \frac{1}{a+b} + \frac{1}{a+b} - \frac{2}{a+b} = 0$ (hence A)

(B) $f(a) + a + b = -(a + b) + (a + b) = 0$ (hence B)
$f(b) + a + b = b^2 - ab - a - b \neq 0$

(D) $f\left(\frac{a}{2}\right) + \frac{a^2}{4} + a + b = \frac{a^2}{4} - a\left(\frac{a}{2} + 1\right) - b + \frac{a^2}{4} + a + b = 0$

8. Let $(x) = x^3 + bx^2 + cx + d$
$b + c + d = 0$ ......(i)
$4b + 2c + d = -4$ ......(ii)
$9b + 3c + d = -18$ ......(iii)
by (i), (ii) and (iii) $b = -5, c = 11, d = -6 \Rightarrow$ $f(x) = x^3 - 5x^2 + 11x - 6$
Alter : $f(x) = (x - 1)(x - 2)(x - 3) + x^2 = x^3 - 5x^2 + 11x - 6 = x^3 - (x - 1)(5x - 6)$

$\Rightarrow$ $f(4) = (3)(2)(1) + 16 = 22$ $f\left(\frac{6}{5}\right) = \left(\frac{6}{5}\right)^3$

Now $f(x) = x^3 \Rightarrow x = 1$ or $\frac{6}{5}$
$f(0)\ f(1) = (-6)(1) < 0$ one root in (0, 1)

9. **Case-I** (i) $x > 1$ $p(x) = x^{25}(x^7 - 1) + x^{11}(x^7 - 1) + x^3(x - 1) + 1$ $p(x) > 0$ no root for $x \in (1, \infty)$
(ii) $0 < x < 1$ $p(x) = x^{32} + x^{18}(1 - x^7) + x^4(x - x^7) + (1 - x^3)$ $p(x) > 0$ not root for (0, 1)
(iii) $x = 1$; $P(x) = 1$
hence no real root for $x > 0$
Case-II : for $x < 0$ let $x = -\alpha$ is root $(\alpha > 0)$ $p(\alpha) = \alpha^{32} + \alpha^{25} + \alpha^{18} + \alpha^{11} + \alpha^4 + \alpha^3 + 1$ $p(\alpha) \neq 0$
Hence no negative rootAll roots are imaginary
$p(x) + p(-x) = 2(x^{32} + x^{18} + x^4 + 1) \neq 0\ \forall\ x \in R$ Hence imaginary roots.

10. $x^2 + px + q = 0 <^{\alpha}_{\beta} \Rightarrow \alpha + \beta = -p,\ \alpha\beta = q$ and $p^2 - 4q > 0 \Rightarrow x^2 - rx + s = 0 <^{\alpha^4}_{\beta^4}$ .....(1)
Now $\alpha^4 + \beta^4 = r \Rightarrow \alpha^4 + \beta^4 = r$, $(\alpha\beta)^4 = s = q^4$ $\therefore$ $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = r$
$\Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = r \Rightarrow (p^2 - 2q)^2 - 2q^2 = r \Rightarrow (p^2 - 2q)^2 = 2q^2 + r > 0$ .....(2)
Now, for $x^2 - 4qx + 2q^2 - r = 0$ $\Rightarrow$
$D = 16q^2 - 4(2q^2 - r)$ by equation (2) $= 8q^2 + 4r = 4(2q^2 + r) > 0 \Rightarrow D > 0$ two real and distinct roots
Product of roots $= 2q^2 - r = 2q^2 - [(p^2 - 2q)^2 - 2q^2] = 4q^2 - (p^2 - 2q)^2 = -p^2(p^2 - 4q) < 0$ from (1)
So product of roots is – ve. hence roots are opposite in sign

11. $ax^3 + bx^2 + cx + d = 0$ with roots α, β, γ
Let $ax^3 + bx^2 + cx + d \equiv (x^2 + x + 1)(Ax + B)$
Roots of $x^2 + x + 1 = 0$ are imaginary, Let these are α, β So the third root 'γ' will be real.
$\alpha + \beta + \gamma = \frac{-b}{a} \Rightarrow -1 + \gamma = \frac{-b}{a} \Rightarrow \gamma = \frac{a-b}{a}$

Also $\alpha\beta\gamma = \frac{-d}{a}$. But $\alpha\beta = 1$

$\therefore$ $\gamma = \frac{-d}{a}$

$\therefore$ Ans are (A) & (D).

12. If $-5 + i\beta$ is a root then other root is $-5 - i\beta$ and $\gamma = 0$

$\Rightarrow$ roots are $-5 + i\beta, -5 - i\beta, -5$

Product of roots $(25 + \beta^2)(-5) = -860$; $25 + \beta^2 = 172$; $\beta^2 = 147$; $\beta = \pm 7\sqrt{3}$

$\therefore$ roots are $-5 + 7i\sqrt{3}, -5 - 7i\sqrt{3}, -5$

and $c = -5(-5 + 7i\beta) - 5(-5 - 7i\sqrt{3}) + (-5 + 7i\sqrt{3})(-5 - 7i\sqrt{3})$

$c = 50 + (250 + 147) = 222.$

13. $f(x) > 0\ \forall x \in R$ or $f(x) < 0\ \forall x \in R$ hence $D < 0$

its graph can be

(i) or (ii)

(A) $f(1) > 0$ graph (i) will be possible so $f(x) > 0\ \forall x \in R$

(B) $f(-1) < 0$ graph (ii) will be possible so $f(x) < 0\ \forall x \in R$

(C) $f\left(-\frac{1}{2}\right) > 0$ so $f(x) < 0\ \forall x \in R$

so not possible

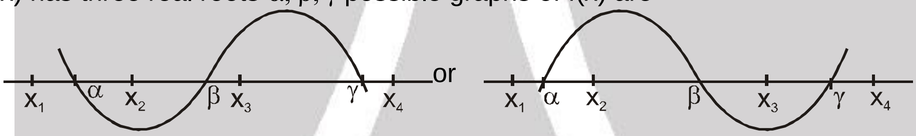
(D) $a > 0 \quad c > 0 \quad$ (graph (i))

$a < 0 \quad c < 0 \quad$ (graph (ii))

in both cases $ac > 0$

14. $f(\alpha) = f(\beta) = \mathbf{f(\gamma) = 0}$

hence f(x) has three real roots α, β, γ possible graphs of f(x) are



$\alpha \in (x_1\ x_2)$, $\beta \in (x_2\ x_3)$ and $\gamma \in (x_3\ x_4)$ or $\alpha \in (x_1\ x_3)$, $\beta \in (x_2\ x_3)$ and $\gamma \in (x_2\ x_4)$

hence A and D are correct

B is wrong as $\beta \notin (x_3, x_4)$

C is wrong as $\beta \notin (x_1, x_2)$

15. only A and C are correct as in these graphs

$f(\alpha) = f(\beta) = f(\gamma) = f'(x_1) = f'(x_2) = 0$

In option B $f(\alpha) < 0$ and $f(\beta) > 0$ (can't be equal).

In option D $f(\alpha) > 0$ and $f(\beta) < 0$ (can't be equal).

16. $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$ $\quad \left.\begin{array}{ll} \because & f(2^+) \to \infty \\ \text{and} & f(3^-) \to -\infty \end{array}\right\} \Rightarrow$ f(x) = 0 has exactly one root in (2, 3).

again $\left.\begin{array}{ll} \because & f(3^+) \to \infty \\ \text{and} & f(4^-) \to -\infty \end{array}\right\}$

$\Rightarrow$ f(x) = 0 has exactly one root in (3, 4).

17. $\because$ D of $x^2 + 4x + 5 = 0$ is less than zero

$\Rightarrow$ both the roots are imaginary

$\Rightarrow$ both the roots of quadratic are same

$\Rightarrow$ $b^2 - 4ac < 0$ & $\frac{a}{1} = \frac{b}{4} = \frac{c}{5} = k$

$\Rightarrow$ $a = k, b = 4k, c = 5k.$

18. $x^2 + abx + c = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$ ...(1) $\alpha + \beta = -ab,\ \alpha\beta = c$

$x^2 + acx + b = 0 \begin{matrix} \alpha \\ \delta \end{matrix}$ ...(2) $\alpha + \delta = -ac,\ \alpha\delta = b$

$\alpha^2 + ab\,\alpha + c = 0$

$\alpha^2 + ac\,\alpha + b = 0$

$$\frac{\alpha^2}{ab^2 - ac^2} = \frac{\alpha}{c - b} = \frac{1}{a\ \ c - ab} \Rightarrow \alpha^2 = \frac{a\ \ (b^2 - c^2)}{a(c - b)} = -(b + c)$$

$$\&\ \ \alpha = \frac{c - b}{a(c - b)} = \frac{1}{a}$$

$\therefore$ common root, $\alpha = \dfrac{1}{a}$

$\therefore\ -(b + c) = \dfrac{1}{a^2} \quad \Rightarrow \quad a^2(b + c) = -1$

Product of the roots of equation (1) & (2) gives

$\beta \times \dfrac{1}{a} = c \Rightarrow \beta = ac \quad \& \quad \delta \times \dfrac{1}{a} = b \Rightarrow \delta = ab.$

$\therefore$ equation having roots β, δ is (β, δ)

$x^2 - a(b + c)x + a^2bc = 0$

$\Rightarrow \quad a(b + c)x^2 - a^2(b + c)^2x + a.(b + c)a^2bc = 0$

$a(b + c)x^2 + (b + c)x - abc = 0.$

19. $\mathbf{S_1}$ **:** $2x^2 + 3x + 1 = 0$

$\because\ D = 9 - 4 \times 1 \times 1 = 1$

Which is perfect square of a rational number

$\therefore$ roots will be rational.

$\mathbf{S_2}$ **:** $\because$ Let $f(x) = (x - a)(x - c) + 2(x - b)(x - d)$

$\because \quad f(a) > 0 \quad f(b) < 0 \quad f(c) < 0 \ f(d) > 0$

$\therefore$ two real and distinct roots.

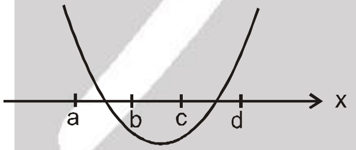
$\mathbf{S_3}$ **:** $x^2 + 3x + 5 = 0$ .....(i) and $ax^2 + bx + c = 0$ .....(ii)

for equation (i) , $D < 0$

$\therefore$ Roots are imaginary and they occur in conjugate pair

$\therefore$ Roots of equation (i) and (ii) will be identical

$\Rightarrow \quad \dfrac{a}{1} = \dfrac{b}{3} = \dfrac{c}{5} = \lambda\ ,\ (\lambda \in N) \Rightarrow \quad a = \lambda,\ \ b = 3\lambda,\ \ c = 5\lambda \Rightarrow a + b + c = 9\lambda\ \therefore$ least value is 9.



20. $x^2 + ax + 12 = 0$ .....(1)

$x^2 + bx + 15 = 0$ .....(2)

$x^2 + (a + b)x + 36 = 0$ .....(3)

(1) + (2) – (3) gives $x^2 - 9 = 0 \quad \Rightarrow \quad x = \pm 3$ given that common root will be +ve

so $x = 3$ put in equation (3) $9 + 3(a + b) + 36 = 0 \quad \Rightarrow \quad a + b = -15$

by equation (1) $9 + 3a + 12 = 0 \Rightarrow a = -7\ \&\ b = -8$

21. $4x^3 + 3x + 2c = (4x + 2c)(x^2 + \lambda x + 1)$

compairing co-efficents $\Rightarrow \quad c = 1$ and $\lambda = -\dfrac{1}{2}$ or $c = -1$ and $\lambda = \dfrac{1}{2}$

$\Rightarrow \quad c + \lambda = \dfrac{1}{2} \quad$ or $\quad -\dfrac{1}{2}$

PART - IV :

1. $x^2 + 2xy + 2y^2 + 4y + 7 = (x + y)^2 + (y + 2)^2 + 3 \geq 0 + 0 + 3$ $\qquad \therefore$ Least value = 3.

2. $P(x) = 4x^2 + 6x + 4 = 4\left(x + \frac{3}{4}\right)^2 + \frac{7}{4} \quad \Rightarrow \quad P(x) = \geq \frac{7}{4}$

$Q(y) = 4y^2 - 12y + 25 = 4\left(y - \frac{3}{2}\right)^2 + 16 \quad \Rightarrow \quad Q(y) \geq 16$

$\Rightarrow$ p(x).Q(y) $\geq$ 28 but it is given P(x).Q(y) = 28 $\quad$ p(x).Q(y) $\geq$ 28 P(x).Q(y) = 28

$\Rightarrow \quad P(x) = \frac{7}{4} \quad \& \quad Q(y) = 16$

$\Rightarrow \quad x = \frac{-3}{4}$, $y = \frac{3}{2}$; $11y - 26x = 11 \times \frac{3}{2} - 26\,\frac{(-3)}{4} = \frac{33}{2} + \frac{39}{2} = \frac{72}{2} = 36$. **Ans.**

(3 & 4)

Let the coordinates of A(α, 0), B(2α, 0), C(0, 2α). Now $y = x^2 + bx + c$ passes through C(0, 2α)

$\therefore$ given equation of curve reduces to $y = x^2 + bx + 2\alpha$. Now it also passes through A & B

$\therefore$ $0 = \alpha^2 + b\alpha + 2\alpha \quad \Rightarrow \quad 0 = \alpha + b + 2$ ..... (i)

& $0 = 4\alpha^2 + 2\alpha b + 2\alpha \quad \Rightarrow \quad 0 = 2\alpha + b + 1$ ..... (ii)

On solving (i) & (ii) for α & b we get $\alpha = 1$, $b = -3$

$\therefore$ given curve is $y = x^2 - 3x + 2$

3. roots of y = 0 are {2, 1}

4. $(\alpha + \beta) \Rightarrow 3$ $(\because \alpha = 2, \beta = 1) \Rightarrow \alpha - \beta) \Rightarrow 1$

$\therefore$ equation whose roots are 3, 1 is $x^2 - 4x + 3 = 0$

(5 to 7)

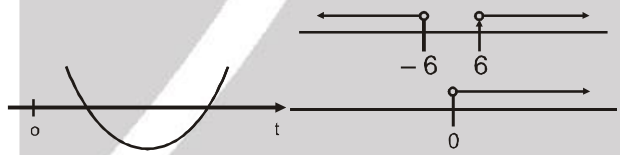
$x^4 - \lambda x^2 + 9 = 0 \Rightarrow \quad x^2 = t \geq 0 \quad \Rightarrow \quad f(t) = t^2 - \lambda t + 9 = 0$

5. given equation has four real & distinct roots

$D > 0 \Rightarrow \lambda^2 - 36 > 0 \quad \frac{-b}{2a} > 0 \Rightarrow \quad \frac{\lambda}{2} > 0 \Rightarrow \lambda > 0$

$f(0) > 0 \Rightarrow \quad 9 > 0$

$\therefore$ $\lambda \in (6,\infty)$



6. Equation has no real roots.

case-I $D \geq 0 \Rightarrow \lambda^2 - 36 \geq 0 \quad \frac{-b}{2a} < 0$

$\Rightarrow \quad \lambda < 0 \quad f(0) > 0$

$\Rightarrow \quad 9 > 0.$

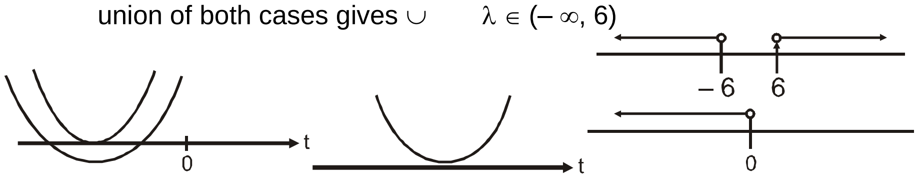
$\therefore \quad \lambda \in (-\infty, -6]$

case-II $D < 0$

$\Rightarrow \quad \lambda^2 - 36 < 0$

$\Rightarrow \quad \lambda \in (-6, 6)$

union of both cases gives $\cup \quad \lambda \in (-\infty, 6)$

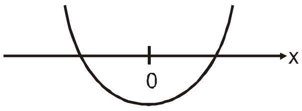


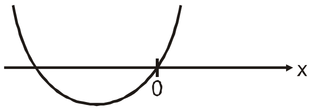
7. Equation has only two real roots

case-I $f(0) < 0$ $9 < 0$

which is false case-II $f(0) = 0$ and $\frac{-b}{2a} < 0$

$\therefore$ No solution $\therefore$ Final answer is ϕ





8. Divide by $x^2 \Rightarrow x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0 \Rightarrow x^2 + \frac{1}{x^2} - 10\left(x + \frac{1}{x}\right) + 26 = 0$

$t = x + \frac{1}{x} \Rightarrow t^2 - 2 = x^2 + \frac{1}{x^2} \Rightarrow t^2 - 2 - 10t + 26 = 0 \Rightarrow t^2 - 10t + 24 = 0 < \begin{matrix} 4 \\ 6 \end{matrix}$

$t = 4 \quad x + \frac{1}{x} = 4 \Rightarrow x^2 - 4x + 1 = 0 \Rightarrow x = 2 \pm \sqrt{3}$

$t = 6 \quad x + \frac{1}{x} = 6 \Rightarrow x^2 - 6x + 1 = 0 \Rightarrow x = 3 \pm 2\sqrt{2}$.

9. By trail $x = 1$ is a root divide by $x - 1$ $\quad x = 1$,

$$\begin{array}{r|cccccc} 1 & 1 & -5 & 9 & -9 & 5 & -1 \\ & \times & 1 & -4 & 5 & -4 & 1 \\ \hline & 1 & -4 & 5 & -4 & 1 & 0 \end{array}$$

$(x - 1)(x^4 - 4x^3 + 5x^2 - 4x + 1) = 0 \Rightarrow x = 1$ or $x^4 - 4x^3 + 5x^2 - 4x + 1 = 0$

$x^2 - 4x + 5 - \frac{4}{x} + \frac{1}{x^2} = 0 \Rightarrow t = x + \frac{1}{x} \Rightarrow t^2 = x^2 + \frac{1}{x^2} + 2$

$t^2 - 2 - 4t + 5 = 0 \Rightarrow t^2 - 4t + 3 = 0 < \begin{matrix} 1 \\ 3 \end{matrix} \Rightarrow x + \frac{1}{x} = 1,\ x + \frac{1}{x} = 3$

$x^2 - x + 1 = 0,\quad x^2 - 3x + 1 = 0 \Rightarrow x = \frac{1 \pm i\sqrt{3}}{2},\ x = \frac{3 \pm \sqrt{5}}{2}$

$\therefore$ roots $1, \frac{1 \pm i\sqrt{3}}{2}, \frac{3 \pm \sqrt{5}}{2}$.

10. Divide by $x^3 \Rightarrow x^3 - 4x + \frac{4}{x} - \frac{1}{x^3} = 0;\ x^3 - \frac{1}{x^3} - 4\left(x - \frac{1}{x}\right) = 0$

Put $t = x - \frac{1}{x} \Rightarrow t^3 = x^3 - 3x^2 \frac{1}{x} + 3x \frac{1}{x^2} - \frac{1}{x^3} = x^3 - 3\left(x - \frac{1}{x}\right) - \frac{1}{x^3}$

$t^3 + 3t = x^3 - \frac{1}{x^3}$

Put in equation above $t^3 + 3t - 4t = 0$

$\Rightarrow t^3 - t = 0$

$\Rightarrow t = 0, 1, -1$

$t^3 + 3t - 4t = 0$

$\Rightarrow t^3 - t = 0$

$\Rightarrow t = 0, 1, -1$

$x - \frac{1}{x} = 0,\ x - \frac{1}{x} = 1,\ x - \frac{1}{x} = -1;\ x = \pm 1,\ x^2 - x - 1 = 0,$

$x^2 + x - 1 = 0$

$x = \pm 1,\ x = \frac{1 \pm \sqrt{5}}{2},\ x = \frac{-1 \pm \sqrt{5}}{2}$.

EXERCISE # 3

PART - I

1. (i) $x^2 - 8kx + 16(k^2 - k + 1) = 0$ $\therefore$ $D = 64 (k^2 - (k^2 - k + 1)) = 64 (k - 1) > 0$

$\Rightarrow$ $k > 1$ ....(1)

(ii) $\frac{b}{2a} - > 4$ $\Rightarrow$ $\frac{8k}{2} > 4$ $\Rightarrow$ $k > 1$ ....(2)

(iii) $f(4) \geq 0$

$\Rightarrow$ $16 - 32k + 16 (k^2 - k + 1) \geq 0$ $\Rightarrow$ $k^2 - 3k + 2 \geq 0$

$\Rightarrow$ $(k - 2) (k - 1) \geq 0$ $\Rightarrow$ $k \leq 1$ or $k \geq 2$ ....(3)

$(1) \cap (2) \cap (3)$. Hence $k = 2$

2. Product = 1

Sum = $\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

Since $\alpha^3 + \beta^3 = q$ $\Rightarrow$ $-p(\alpha^2 + \beta^2 - \alpha\beta) = q$

$((\alpha + \beta)^2 - 3\alpha\beta) = -\frac{q}{p}$ $\Rightarrow$ $p^2 + \frac{q}{p} = 3\alpha\beta$

Hence sum = $\frac{\left\{p^2 - \frac{2}{3}\left(\frac{p^3+q}{p}\right)\right\} 3p}{(p^3+q)} = \frac{p^3-2q}{p^3+q}$

so the equation $x^2 - \left(\frac{p^3-2q}{p^3+q}\right) x + 1 = 0$ $\Rightarrow$ $(p^3 + q) x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

3. $x^2 - 6x - 2 = 0$ having roots α and β $\Rightarrow$ $\alpha^2 - 6\alpha - 2 = 0$

$\Rightarrow$ $\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ $\Rightarrow$ $\alpha^{10} - 2\alpha^8 = 6\alpha^9$ .... (i)

similarly $\beta^{10} - 2\beta^8 = 6\beta^9$ .... (ii)

by (i) and (ii)

$(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8) = 6(\alpha^9 - \beta^9)$ $\Rightarrow$ $a_{10} - 2a_8 = 6a_9$ $\Rightarrow$ $\frac{a_{10} - 2a_8}{2a_9} = 3$

Aliter

$$\frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^{10} - \beta^{10} + \alpha\beta(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^9(\alpha+\beta) - \beta^9(\alpha+\beta)}{2(\alpha^9 - \beta^9)} = \frac{\alpha+\beta}{2} = \frac{6}{2} = 3$$

4. $\begin{array}{l} x^2 + bx - 1 = 0 \\ x^2 + x + b = 0 \\ \hline \frac{x^2}{b^2+1} = \frac{x}{-1-b} = \frac{1}{1-b} \end{array}$ $\Rightarrow$ $x = \frac{b^2+1}{-(b+1)} = \frac{-(b+1)}{1-b}$ $\Rightarrow$ $(b^2+1)(1-b) = (b+1)^2$

$\Rightarrow$ $b^2 - b^3 + 1 - b = b^2 + 2b + 1$ $\Rightarrow$ $b^3 + 3b = 0 \Rightarrow b = 0$; $b^2 = -3 \Rightarrow b = 0 \pm \sqrt{3}$ i ,

5. p(x) will be of the form $ax^2 + c$. Since it has purely imaginary roots only.

Since p(x) is zero at imaginary values while $ax^2 + c$ takes real value only at real 'x', no root is real.

Also $p(p(x)) = 0$

$\Rightarrow$ p(x) is purely imaginary $\Rightarrow$ $ax^2 + c$ = purely imaginary

Hence x can not be purely imaginary since x^2 will be negative in that case and $ax^2 + c$ will be real.

Thus .(D) is correct.

6. $(x_1 + x_2)^2 - 4x_1x_2 < 1$ $\frac{1}{\alpha^2} - 4 < 1 \Rightarrow 5 - \frac{1}{\alpha^2} > 0$ $\frac{5\alpha^2 - 1}{\alpha^2} > 0$

$$\begin{array}{cccc} + & - & - & + \\ \frac{1}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} & \end{array}$$

$\alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ ...(1)

$D > 0$ $1 - 4\alpha^2 > 0$ $\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ ...(2)

(1) & (2) $\alpha \in \left(-\frac{1}{2}, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$

7. $x^2 - 2x\sec\theta + 1 = 0 \Rightarrow x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} \Rightarrow x = \sec\theta + \tan\theta$, $\sec\theta - \tan\theta \Rightarrow \alpha_1 = \sec\theta - \tan\theta$

now $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = \frac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$ $\Rightarrow x = -\tan\theta \pm \sec\theta \Rightarrow$ $\alpha_2 = (\sec\theta - \tan\theta)$

$\Rightarrow$ $\beta_2 = -(\sec\theta + \tan\theta)$

$\therefore$ $\alpha_1 + \beta_2 = -2\tan\theta$

Alt : (i) $x^2 - 2x\sec\theta + 1 = 0$ $x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} = \sec\theta \pm \tan\theta$

$\alpha_1 = \sec\theta - \tan\theta$ $\beta_1 = \sec\theta + \tan\theta$

(ii) $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = \frac{-\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$x = -\tan\theta \pm \sec\theta$ $\alpha_2 = -\tan\theta + \sin\theta$ $\beta_2 = -\tan\theta - \sec\theta$ $\alpha_1 + \beta_2 = -2\tan\theta$

8. As α and β are roots of equation $x^2 - x - 1 = 0$, we get : $\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^2 = \alpha + 1$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^2 = \beta + 1$

$\therefore a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10} = p\alpha^{10}(\alpha+1) + q\beta^{10}(\beta+1) = p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2 = p\alpha^{12} + q\beta^{12} = a_{12}$

9. $a_{n+2} = a_{n+1} + a_n$ $a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

As $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$, we get $a_4 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$

$\Rightarrow \left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q - 28\right) = 0$ ....(i)

and $\frac{3p}{2} - \frac{3q}{2} = 0$ ....(ii)

$\Rightarrow$ p = q (from (ii)) $\Rightarrow$ 7p = 28 (from (i) and (ii))

$\Rightarrow$ p = 4 $\Rightarrow$ q = 4 $\Rightarrow$ p + 2q = 12

10. (A) $b_n = a_{n+1} + a_{n-1} = \dfrac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta} + \dfrac{\alpha^{n-1}-\beta^{n-1}}{\alpha-\beta} = \dfrac{\alpha^{n-1}(\alpha^2+1)-\beta^{n-1}(\beta^2+1)}{\alpha-\beta}$

$= \dfrac{\alpha^{n-1}(\alpha+2)-\beta^{n-1}(\beta+2)}{\alpha-\beta} = \dfrac{\alpha^{n-1}\left(\dfrac{5+\sqrt{5}}{2}\right)-\beta^{n-1}\left(\dfrac{5-\sqrt{5}}{2}\right)}{\alpha-\beta}$

$= \dfrac{\sqrt{5}\alpha^{n-1}\left(\dfrac{\sqrt{5}+1}{2}\right)-\sqrt{5}\beta^{n-1}\left(\dfrac{\sqrt{5}-1}{2}\right)}{\alpha-\beta} = \dfrac{\sqrt{5}(\alpha^n+\beta^n)}{\alpha-\beta} = \alpha^n + \beta^n \quad \because \quad \alpha - \beta = \sqrt{5}$

(B) $\sum_{n=1}^{\infty}\dfrac{b_n}{10^n} = \sum\left(\dfrac{\alpha}{10}\right)^n + \sum\left(\dfrac{\beta}{10}\right)^n = \dfrac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \dfrac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \dfrac{\alpha}{10-\alpha}+\dfrac{\beta}{10-\beta}$

$= \dfrac{10(\alpha+\beta)-2\alpha\beta}{100-10(\alpha+\beta)+\alpha\beta} = \dfrac{10+2}{89} = \dfrac{12}{89}$

(C) $\sum_{n=1}^{\infty}\dfrac{a_n}{10^n} = \sum\dfrac{\alpha^n-\beta^n}{(\alpha-\beta)10^n} = = \dfrac{1}{\alpha-\beta}\left(\dfrac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1-\frac{\beta}{10}}\right)\dfrac{1}{\alpha-\beta}\left(\dfrac{\alpha}{10-\alpha}-\dfrac{\beta}{10-\beta}\right)$

$= \dfrac{1}{\alpha-\beta}.\dfrac{(10(\alpha-\beta)-\alpha\beta+\alpha\beta)}{100-10(\alpha+\beta)+\alpha\beta} = \dfrac{10}{89}$ Option (C) is correct.

(D) $a_1 + a_2 + \ldots.. a_n = \Sigma a_i = \dfrac{\Sigma\alpha^i - \Sigma\beta^i}{\alpha-\beta} = \dfrac{\dfrac{\alpha(1-\alpha^n)}{(1-\alpha)} - \dfrac{\beta(1-\beta^n)}{(1-\beta)}}{\alpha-\beta}$

$= \dfrac{(\alpha+1)(1-\alpha^n)-(\beta+1)(1-\beta^n)}{(1-\alpha)(1-\beta)(\alpha-\beta)} = \dfrac{\alpha^2-\alpha^{n+2}-\beta^2+\beta^{n+2}}{(1-\alpha)(1-\beta)(\alpha-\beta)} = \dfrac{\sqrt{5}+\beta^{n+2}-\alpha^{n+2}}{\beta-\alpha} = -1 + a_{n+2}$

PART - II

1. Let the correct equation be $ax^2 + bx + c = 0$ $ax^2 + bx + c = 0$

now sachin's equation $\Rightarrow$ $ax^2 + bx + c' = 0$ $\begin{cases}\nearrow 4\\ \searrow 3\end{cases}$

Rahul's equation $\Rightarrow$ $ax^2 + b'x + c = 0$ $\begin{cases}\nearrow 3\\ \searrow 2\end{cases}$

$-\dfrac{b}{a} = 7$ .... (i) $\dfrac{c}{a} = 6$ .... (ii)

from (i) and (ii)

correct equation is $x^2 - 7x + 6 = 0$ roots are 6 and 1. $x^2 - 7x + 6 = 0$

2. $P(x) = 0 \Rightarrow$ $f(x) = g(x) \Rightarrow$ $ax^2 + bx + c = a_1x^2 + b_1x + C, \Rightarrow (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0.$

It has only one solution $x = -1$

$\Rightarrow$ $b - b_1 = a - a_1 + c - c_1$ .... (1)

vertex $(-1, 0)$ $\Rightarrow$ $\dfrac{b-b_1}{2(a-a_1)} = -1$

$\Rightarrow$ $b - b_1 = 2(a - a_1)$ .... (2)

$\Rightarrow$ $f(-2) - g(-2) = 2 \Rightarrow 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$

$\Rightarrow$ $4(a - a_1) - 2(b - b_1) + (c - c_1) = 2$ .... (3)

by (1), (2) and (3) $(a - a_1) = (c - c_1) = \dfrac{1}{2}$ $(b - b_1) = 2$

Now $P(2) = f(2) - g(2) \quad = 4(a - a_1) + 2(b - b_1) + (c - c_1) = 8 + 8 + 2 = 18$

3. Let $e^{\sin x} = t \quad \Rightarrow \quad t^2 - 4t - 1 = 0$

$\Rightarrow \quad t = \dfrac{4 \pm \sqrt{16+4}}{2}$

$\Rightarrow \quad t = e^{\sin x} = 2 \pm \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 - \sqrt{5}\ , \quad e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 - \sqrt{5} < 0,$

$\Rightarrow \quad \sin x = ln(2 + \sqrt{5}) > 1$ so rejected so rejected hence no solution

4. $x^2 + 2x + 3 = 0 \quad$ ...(i)

$ax^2 + bx + c = 0 \quad$ ...(ii)

Since equation (i) has imaginary roots.

So equation (ii) will also have both roots same as (i).

Thus $\dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3} \Rightarrow \quad a = \lambda,\ b = 2\lambda,\ c = 3\lambda$. Hence 1 : 2 : 3

5. $a^2 = 3\{x\}^2 - 2\{x\}$ $\qquad$ [$x - [x] = \{x\}$]

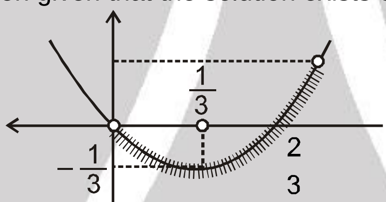
Let $\{x\} = t \quad \therefore \quad t \in (0, 1)$ $\qquad$ As x is not an integer

$\therefore \quad a^2 = 3t^2 - 2t \quad f(t) = 3t\left(t - \dfrac{2}{3}\right) \Rightarrow \quad a^2 = 3t\left(t - \dfrac{2}{3}\right)$

Clearly by graph $\qquad -\dfrac{2}{3} \le a^2 < 1$

$\therefore \quad a \in (-1, 1) - \{0\}$ $\qquad$ (As $x \ne$ integer) **Ans.** (3)

Note : It should have been given that the solution exists else answer will be $a \in R - \{0\}$



6. $px^2 + qx + r = 0 \genfrac{}{}{0pt}{}{\alpha}{\beta}$; $\quad$ p, q, r $\rightarrow$ A.P. ; $\quad 2q = p + r \quad \dfrac{1}{\alpha} + \dfrac{1}{\beta} = 4$; $\quad \dfrac{\alpha+\beta}{\alpha\beta} = 4 \Rightarrow \quad \dfrac{-q}{r} = 4$

$q = -4r \quad$ ..... (i)

$\therefore \quad -8r = p + r$

$p = -9r \quad$ ..... (ii)

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\dfrac{q^2}{p^2} - \dfrac{4r}{p}}$ by (i) and (ii)

$= \dfrac{\sqrt{q^2 - 4pr}}{|p|} = \dfrac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \dfrac{2\sqrt{13}}{9}$

7. $x^2 - 6x - 2 = 0 \quad a_n = \alpha^n - \beta^n$

$\dfrac{a_{10} - 2a_8}{2a_9} = \dfrac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \dfrac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} = \dfrac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)} \quad \dfrac{\alpha+\beta}{2} = \dfrac{6}{2} = 3$ $\quad$ Ans. (3)

8. For rational roots D must be perfect square $D = 121 - 24\alpha = k^2$ for $121 - 24\alpha$ to be perfect square α must be equal to 3, 4, 5 (observation) so number of possible values of α is 3.

9. Let roots are α & β now $\lambda + \dfrac{\lambda}{\lambda} = 1 \Rightarrow \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = 1 \Rightarrow \alpha^2 + \beta^2 = \alpha\beta \quad (\alpha+\beta)^2 = 3\alpha\beta \quad \left(\dfrac{-m(m-4)}{3m^2}\right)^2 = 3.\dfrac{2}{3m^2}$

$m^2 - 8m - 2 = 0 \quad m = 4 \pm 3\sqrt{2}$ $\quad$ so least value of $m = 4 - 3\sqrt{2}$

HIGH LEVEL PROBLEMS (HLP)

1. $\alpha_1^3\dfrac{(x-\alpha_2)(x-\alpha_3)......(x-\alpha_n)}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)......(\alpha-\alpha_n)}+\dfrac{(x-\alpha_1)(x-\alpha_3)......(x-\alpha_n)}{(\alpha_2-\alpha_1)(\alpha_2-\alpha_3)......(\alpha_2-\alpha_n)}\alpha_2^3+..........$

$........+\dfrac{(x-\alpha_1)(x-\alpha_2)......(x-\alpha_{n-1})}{(\alpha_n-\alpha_1)(\alpha_n-\alpha_2)......(\alpha_n-\alpha_{n-1})}\alpha_n^3-x^3=0$

$\Rightarrow$ $\alpha_1, \alpha_2 ....\alpha_n$ are roots of above relation whose degree appeared as (n–1)

$\Rightarrow$ above relation is indentify

2. $a^2x^2 + (b^2 + a^2 - c^2)\,x + b^2 = 0$ ......(1)

$\because$ $a + b > c \Rightarrow a + b - c > 0$ ......(2)

and $|a - b| < c \Rightarrow$ $a - b - c < 0$ ....(3)

and $a - b + c > 0$ ....(4)

Discriminant of equation (1) i.e. $D = (b^2 + a^2 - c^2)^2 - 4a^2b^2 = (b^2 + a^2 - c^2 - 2ab)\,(b^2 + a^2 - c^2 + 2ab)$

$= \{(a - b)^2 - c^2\}\,\{(a + b)^2 - c^2\}$

$= (a - b + c)\,(a - b - c)\,(a + b + c)\,(a + b - c) < 0$ (using (2), (3), (4))

$D < 0$

$\therefore$ roots are not real.

3. $\dfrac{1}{x-1}-\dfrac{4}{x-2}+\dfrac{4}{x-3}-\dfrac{1}{x-4}<\dfrac{1}{30}$

$\Rightarrow$ $\dfrac{-3}{x^2-5x+4}+\dfrac{4}{x^2-5x+6}<\dfrac{1}{30}$

Let $x^2 - 5x = y$

$\Rightarrow$ $\dfrac{4}{y+6}-\dfrac{3}{y+4}<\dfrac{1}{30}$

$\Rightarrow$ $\dfrac{y^2-20y+84}{(y+6)(y+4)}>0$

$\Rightarrow$ $y \in (-\infty, -6) \cup (-4, 6) \cup (14, \infty)$

Now (i) if $y \in (-\infty, -6)$

$\Rightarrow$ $x^2 - 5x < -6.$

$\Rightarrow$ $x \in (2, 3)$

(ii) if $y \in (-4, 6)$ $\Rightarrow$ $-4 < x^2 - 5x < 6$

$\Rightarrow$ $x \in (-1, 1) \cup (4, 6)$

(iii) if $y \in (14, \infty)$ $\Rightarrow$ $x^2 - 5x > 14$

$\Rightarrow$ $x \in (-\infty, -2) \cup (7, \infty)$

$\therefore$ final answer is $x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$

4. Let the three numbers in G.P. be a, ar, ar^2

$\because$ $a + b + c = xb$

$\dfrac{a}{b}+1+\dfrac{c}{b}=x$ $\because$ $b = ar,\ c = ar^2$

$\Rightarrow$ $r^2 + (1 - x)r + 1 = 0$ ....(1)

r is real $\therefore$ for (1) $D \ge 0$

$\Rightarrow$ $(1 - x)^2 - 4 \ge 0$

$\Rightarrow$ $x^2 - 2x - 3 \ge 0$

$\Rightarrow$ $x \le -1$ or $x \ge 3$

Note: If we put $x = -1$ and $x = 3$ in (1) we get $r = -1$ and $r = 1$ respectively which is not possible because in both cases the three numbers will not be distinct therefore $x < -1$ or $x > 3$

5. $V_n + V_{n-3} = (\alpha^n + \beta^n) + (\alpha^{n-3} + \beta^{n-3}) = \alpha^{n-3}(\alpha^3 + 1) + \beta^{n-3}(\beta^3 + 1)$

$\because \quad \alpha^2 + \alpha - 1 = 0$

$$\begin{array}{r|l} \alpha^2+\alpha-1 & \alpha^3 + 1 \quad (\alpha - 1 \\ & \underline{\alpha^3 + \alpha^2 - \alpha} \\ & \;\;-\quad\;\; -\quad\;\; + \\ & -\alpha^2 + \alpha + 1 \\ & \underline{-\alpha^2 - \alpha + 1} \\ & \;\;+\quad\;\; +\quad\;\; - \\ & \;\; 2\alpha \end{array}$$

$\alpha^3 + 1 = (\alpha^2 + \alpha - 1)(\alpha - 1) + 2\alpha = 2\alpha$

also $\beta^3 + 1 = 2\beta \quad \Rightarrow \quad V_n + V_{n-3} = \alpha^{n-3}(2\alpha) + \beta^{n-3}(2\beta) = 2[\alpha^{n-2} + \beta^{n-2}] = 2V_{n-2}$

$V_1 = \alpha + \beta = -1$; $V_2 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2(-1) = 3$

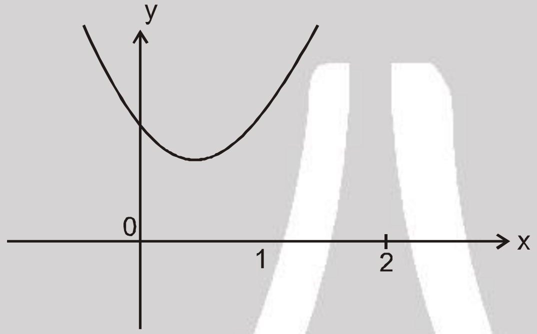
$V_n = 2V_{n-2} - V_{n-3} \Rightarrow \quad V_7 = 2V_5 - V_4 = 2[2V_3 - V_2] - (2V_2 - V_1) = 4V_3 - 2V_2 - 2V_2 + V_1$

$= 4[2V_1 - V_0] - 4V_2 + V_1 = 9V_1 - 4V_0 - 4V_2 = 9\{[-1]\} - 4[2] - 4[3] = -9 - 8 - 12 = -29$

6. $f(x) = x^2 - (m - 3)x + m > 0 \;\forall x \in [1, 2]$. Here $D = (m - 3)^2 - 4m = m^2 - 10m + 9 = (m - 1)(m - 9)$

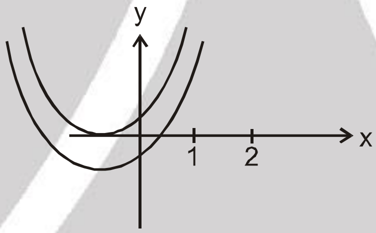
All possible graphs are

Case 1 :



$D < 0 \Rightarrow \quad m \in (1, 9)$

Case 2 :



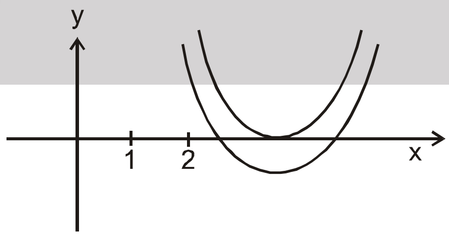
(i) $f(1) > 0 \quad \Rightarrow \quad 4 > 0$ always true

(ii) $-\frac{b}{2a} < 1 \quad \Rightarrow \quad m < 5$

(iii) $D \geq 0 \quad \Rightarrow \quad m \in (-\infty, 1]\;\; [9, \infty)$

$\therefore \quad$ (i) $\cap$ (ii) $\cap$ (iii), we get $m \in (-\infty, 1]$

Case 3 :



(i) $f(2) > 0 \quad \Rightarrow \quad m < 10$

(ii) $\frac{-b}{2a} > 2 \quad \Rightarrow \quad m > 7$

(iii) $D \geq 0 \quad \Rightarrow \quad m \in (-\infty, 1] \cup [9, \infty)$

$\therefore \quad$ (i) $\cap$ (ii) $\cap$ (iii), we get $m \in [9, 10)$

Now final Answer is (Case 1) $\cup$ (Case 2) $\cup$ (Case 3) we get $m \in (-\infty, 10)$

7. Let $f(x) = ax^2 + (a-2)x - 2$ $\because$ $f(0) = -2$ and $f(-1) = 0$

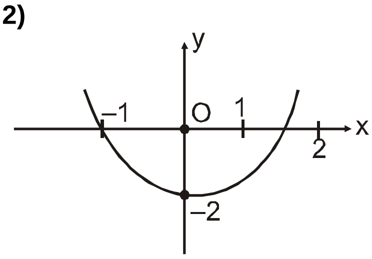
Since the quadratic expression is negative for exactly two integral values

$\Rightarrow$ $f(1) < 0$ and $f(2) \geq 0$

$\Rightarrow$ $a + a - 2 - 2 < 0$ and $4a + 2a - 4 - 2 \geq 0$

$\Rightarrow$ $a < 2$ and $a \geq 1$

$\therefore$ $\mathbf{a \in [1, 2)}$



8. **(i)** when $x < a \Rightarrow x^2 + 2a(x-a) - 3a^2 = 0 \Rightarrow (x+a)^2 = 6a^2$

$x = -a \pm \sqrt{6}a = -a(1+\sqrt{6}), -a$. Since $a \leq 0$, then $x = -a(1-\sqrt{6})$

when $x \geq a$ then $x^2 - 2a(x-a) - 3a^2 = 0 \Rightarrow x = a \pm \sqrt{2}a = a((1+\sqrt{2}), a(1-\sqrt{2})$

since $a \leq 0$, $x \geq a$

$\therefore$ $x = a(1-\sqrt{2})$ Hence $x = a(\sqrt{6}-1), a(1-\sqrt{2})$

(iii) $\left(x+\frac{1}{x}\right)^3 + \left(x+\frac{1}{x}\right) = 0 \Rightarrow \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2 + 1\right) = 0 \Rightarrow \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+3\right) = 0$

$\Rightarrow$ No real value of x. $\therefore$ Number of real roots = 0

9. $\because$ α, β are the roots of $x^2 - 34x + 1 = 0$ $\Rightarrow$ $\alpha + \beta = 34$ and $\alpha\beta = 1$

$\because$ $\left(\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}}\right)^2 = \sqrt{\alpha} + \sqrt{\beta} - 2(\alpha\beta)^{1/4}$ $\because$ $\alpha\beta = 1$

$\therefore$ $\left(\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}}\right)^2 = \sqrt{\alpha} + \sqrt{\beta} - 2$ ....(1)

$\because$ $\left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 = \alpha + \beta + 2\sqrt{\alpha\beta}$ $\because$ $\alpha + \beta = 34$ and $\alpha\beta = 1$

$\therefore$ $\left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 = 36$ $\because$ we consider the principal value

$\therefore$ $\sqrt{\alpha} + \sqrt{\beta} = 6$ put in (1), we get. $\Rightarrow$ $\left(\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}}\right)^2 = 4$

$\therefore$ $\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}} = \pm 2$ **Ans.**

10. The equation can be rewritten as $\left(\frac{x^2+x+2}{x^2+x+1}\right)^2 - (a-3)\left(\frac{x^2+x+2}{x^2+x+1}\right) + (a-4) = 0$

Let $\frac{x^2+x+2}{x^2+x+1} = t$ or $t = 1 + \frac{1}{x^2+x+1}$ since $x^2 + x + 1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

Therefore $(x^2 + x + 1) \geq \frac{3}{4} \Rightarrow t \in \left(1, \frac{7}{3}\right]$ Now given equation reduces to $t^2 - (a-3)t + (a-4) = 0$

Atleast one root of the equation must lie in $\left(1, \frac{7}{3}\right]$ Now, $t = \frac{(a-3) \pm \sqrt{(a-3)^2 - 4(a-4)}}{2} \Rightarrow t = a - 4, 1$

For one root to lie in $\left(1, \frac{7}{3}\right]$ we must have $1 < a - 4 \le \frac{7}{3} \Rightarrow 5 < a \le \frac{19}{3}$

11. Let roots be $\alpha, \beta \Rightarrow \alpha\beta = -14(q^2 + 1)$. Clearly, $q^2 + 1$ is not multiple of 7

$\because$ α, β are integers clearly one of α or β is multiple of 7

$\because$ $\alpha + \beta = -7$; which is possible if α, β are both multiple of 7. Hence α, β are not integers.

12. Let $x^2 = t \ge 0$, for only real solution. Again let $f(t) = t^2 - (a^2 - 5a + 6)\, t - (a^2 - 3a + 2)$

$f(0) \ge 0 \Rightarrow a^2 - 3a + 2 \le 0$ $a \in [1, 2] - b/2a \ge 0 \Rightarrow a^2 - 5a + 6 \ge 0 \Rightarrow (a-2)(a-3) \ge 0 \Rightarrow a \in (-\infty, 2] \cup [3, \infty)$

So possible 'a' from above two conditions are a = 1, 2. Now condition for $D = ((a-2)(a-3))^2 + 4(a-1)(a-2) \ge 0$ is also satisfied by these two possible values of a. So required value of 'a' are 1, 2

13. α, β are the roots of $a_1x^2 + b_1x + c_1 = 0 \Rightarrow \alpha + \beta = -\frac{b_1}{a_1}$ and $\alpha\beta = \frac{c_1}{a_1}$

$1 + \alpha + \beta + \alpha\beta = \frac{c_1 - b_1 + a_1}{a_1} \Rightarrow (1+\alpha)(1+\beta) = \frac{a_1 - b_1 + c_1}{a_1}$ ....(1)

Similarly $(1+\beta)(1+\gamma) = \frac{a_2 - b_2 + c_2}{a_2}$ ....(2)

$(1+\gamma)(1+\alpha) = \frac{a_3 - b_3 + c_3}{a_3}$ ....(3)

Multiplying (1), (2) & (3), we get $(1+\alpha)^2(1+\beta)^2(1+\gamma)^2 = \frac{(a_1 - b_1 + c_1)}{a_1}\frac{(a_2 - b_2 + c_2)}{a_2}\frac{(a_3 - b_3 + c_3)}{a_3}$

$$\Rightarrow (1+\alpha)(1+\beta)(1+\gamma) = \left\{\prod_{i=1}^{3}\left(\frac{a_i - b_i + c_i}{a_i}\right)\right\}^{1/2}$$

$$\Rightarrow 1 + (\alpha+\beta+\gamma) + (\alpha\beta + \beta\gamma + \alpha\gamma) + \alpha\beta\gamma = \left\{\prod_{i=1}^{3}\left(\frac{a_i - b_i + c_i}{a_i}\right)\right\}^{1/2}$$

$$\Rightarrow (\alpha+\beta+\gamma) + (\alpha\beta + \beta\gamma + \alpha\gamma) + \alpha\beta\gamma = \left\{\prod_{i=1}^{3}\left(\frac{a_i - b_i + c_i}{a_i}\right)\right\}^{1/2} - 1$$

14. On putting the value of p, q and r in given equation we have

$2x^3 - (a_1 + a_2 \,........ + a_6)x^2 + (a_1a_3 + a_3a_5 \,....... + a_6a_2)x - (a_1a_3a_5 + a_2a_4a_6) = 0$

$\{x^3 - (a_1 + a_3 + a_5)x^2 + (a_1a_3 + a_3a_5 + a_5a_1)x - a_1a_3a_5\} +$

$\{x^3 - (a_2 + a_4 + a_6)x^2 + (a_2a_4 + a_4a_6 + a_6a_2)x - a_2a_4a_6\} = 0$

$\Rightarrow (x - a_1)(x - a_3)(x - a_5) + (x - a_2)(x - a_4)(x - a_6) = 0$

Let $f(x) = (x - a_1)(x - a_3)(x - a_5) + (x - a_2)(x - a_4)(x - a_6)$

Now $f(a_1) = (a_1 - a_2)(a_1 - a_4)(a_1 - a_6) > 0$; $f(a_2) = (a_2 - a_1)(a_2 - a_3)(a_2 - a_5) < 0$

$f(a_3) = (a_3 - a_2)(a_3 - a_4)(a_3 - a_6) < 0$; $f(a_4) = (a_4 - a_1)(a_4 - a_3)(a_4 - a_5) > 0$

$f(a_5) = (a_5 - a_2)(a_5 - a_4)(a_5 - a_6) > 0$; $f(a_6) = (a_6 - a_1)(a_6 - a_3)(a_6 - a_5) < 0$

From above results it is clear that there are three real roots lying in the intervals (a_1, a_2), (a_3, a_4) and (a_5, a_6)

15. Let A_1, A_2 are the roots of $ax^2 + bx + c = 0$, then $(A_1 - A_2)^2 = (A_1 + A_2)^2 - 4A_1A_2 = \frac{b^2}{a^2} - \frac{4c}{a} = \frac{b^2 - 4ac}{a^2}$

Using same result for $x^2 + 2bx + c = 0 \Rightarrow \left\{\left(\beta + \cos^2\alpha\right) - \left(\beta + \sin^2\alpha\right)\right\}^2 = 4b^2 - 4c$

$\Rightarrow \quad \left(\cos^2\alpha - \sin^2\alpha\right)^2 = 4b^2 - 4c \Rightarrow \cos^2 2\alpha = 4(b^2 - c)$ ....(i)

Similarly for $X^2 + 2BX + C = 0 \Rightarrow \left\{\left(\gamma + \cos^4\alpha\right) - \left(\gamma + \sin^4\alpha\right)\right\}^2 = \frac{4B^2 - 4C}{1} = 4B^2 - 4C$

$\Rightarrow \quad \left\{\left(\cos^2\alpha - \sin^2\alpha\right)\left(\cos^2\alpha + \sin^2\alpha\right)\right\}^2 = 4(B^2 - C)$

$\Rightarrow \quad \cos^2 2\alpha = 4(B^2 - C)$ ....(ii)

$\therefore$ from (i) & (ii)

$B^2 - C = b^2 - c$; $B^2 - b^2 = C - c$ Hence Proved.

16. $(x^2 + x)^2 + a(x^2 + x) + 4 = 0$. Let $x^2 + x = t$ then $x^2 + x - t = 0$ $\forall\, x \in R$

$D \geq 0 \Rightarrow 1 + 4t \geq 0 \Rightarrow t \in \left[-\frac{1}{4}, \infty\right)$ ....(1)

Now $f(t) = t^2 + at + 4 = 0$

(i) all four real and distinct roots

(A) $D > 0$

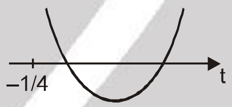
(B) $f(-1/4) > 0$

(C) $-\frac{b}{2a} > -\frac{1}{4}$

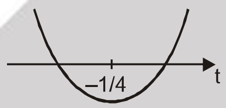
(A) $D > 0 \Rightarrow a^2 - 16 > 0 \Rightarrow |a| > 4$

(B) $f(-1/4) = \frac{1}{16} - \frac{a}{4} + 4 > 0 \Rightarrow a < 65/4$

(C) $-\frac{b}{2a} = -\frac{a}{2} > -\frac{1}{4} \Rightarrow a < \frac{1}{2} \Rightarrow a \in (-\infty, -4)$



(ii) Two real roots which are distinct



$f(-1/4) < 0 \Rightarrow a > 65/4 \Rightarrow a \in (65/4, \infty)$

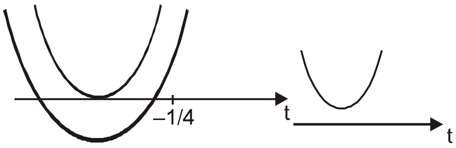
(iii) all four roots are imaginary

Case-(i) (A) $D \geq 0 \Rightarrow |a| \geq 4$

(B) $f(-1/4) > 0 \Rightarrow a < \frac{65}{4}$

(C) $-\frac{b}{2a} < -\frac{1}{4} \Rightarrow a > \frac{1}{2}$

$\Rightarrow a \in \left[4, \frac{65}{4}\right)$



case- (i) case- (ii)

Case-(ii) D < 0 $\Rightarrow a \in (-4, 4)$ ...(i)

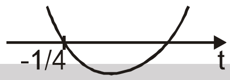
taking union of both conditions mentioned by graph of case-(i) and case-(ii) $a \in \left(-4 \ , \ \frac{65}{4}\right)$

(iv) four real roots in which two are equal

(A) D > 0 (B) f(–1/4) = 0 (C) $-\frac{b}{2a} > -\frac{1}{4}$

(A) |a| > 4 (B) a = 65/4 (C) $a < \frac{1}{2}$

No common solution $\therefore$ $a \in \phi$



17. $(x^2 + bx + c).P(x) = 3x^4 + 18x^2 + 75$ $\Rightarrow$ $(x^2 + bx + c).Q(x) = 3x^4 + 4x^2 + 28x + 5$
equation (i) – (ii) $(x^2 + bx + c)P(x) - Q(x) = 14x^2 - 28x + 70 = 14(x^2 - 2x + 5)$
$x^2 + bx + c = x^2 - 2x + 5$. hence $f(x) = x^2 - 2x + 5$.

18. Because discrminent of $x^2 + (2–a) x + 3 = 0$ is negative so
$x^2 + (2–a) x + 3 = 0$ has both imaginary roots
$\Rightarrow$ both the equation have two common roots
Now, roots of equation $x^2 + (2–a) x + 3 = 0$ are roots of $(ax^4 + bx^3+x^2+(3–a) x+3)–(x^2 + (2–a) x + 3) = 0$
$\Rightarrow$ roots of equation $x^2 + (2–a)x + 3 = 0$ are roots of $ax^4 + bx^3 + x = 0$
$\Rightarrow$ roots of equation $x^2 + (2–a)x + 3 = 0$ are roots of $ax^3 + bx^2 + 1 = 0$
$\Rightarrow$ roots of equation $x^2 + (2–a)x + 3 = 0$ are roots of $(ax^3 + bx^2 +1) – (ax(x^2+(2–a) x + 3)) = 0$
$\Rightarrow$ roots of equation $x^2 + (2–a) x + 3 = 0$ and $(b –2a + a^2) x^2 – 3ax + 1 = 0$ are same

$\Rightarrow$ $\frac{b-2a+a^2}{1} = \frac{2-a}{-3a} = \frac{3}{1}$ $\Rightarrow a = -\frac{1}{4}$ and $b = -\frac{11}{48}$

$\Rightarrow$ $|a + 12b| = 3$

19. $\alpha\beta = \alpha^2\beta^2$ ......(1) $\alpha + \beta = \alpha^2 + \beta^2$ ......(2)
From (1), $\alpha\beta = 0$ or $\alpha\beta = 1$
Case 1 : If $\alpha\beta = 0$ then we have following possibilities
(i) If $\alpha = 0$ then from (2), $\beta = 0$ or $\beta = 1$
(ii) If $\beta = 0$ then from (2), $\alpha = 0$ or $\alpha = 1$
$\therefore$ from (i) and (ii) we can say that roots are $\alpha = 0, \beta = 0$ or $\alpha = 0, \beta = 1$
$\therefore$ Required quadratic equations are $x^2 = 0$ or $x^2 – x = 0$
Case 2 : If $\alpha\beta = 1$ then from (2) we get $\alpha + \beta = \alpha^2 + \beta^2 = (\alpha + \beta)^2 – 2\alpha\beta$ $\because \alpha\beta = 1$
$\therefore$ $\alpha + \beta = –1, 2$ So the required quadratic equations are $x^2 + x + 1 = 0$
and $x^2 – 2x + 1 = 0$ $\Rightarrow$ four equations are possible.

20. Let $y = Q(x) \Rightarrow$ $\sqrt{y+2} = x$ $\Rightarrow$ $P\left(\sqrt{y+2}\right) = 0$

$(y+2)^{\frac{5}{2}} = -y-3.$ $\Rightarrow$ $y^5 + 10y^4 + 40y^3 + 79y^2 + 74y + 23 = 0.$

(i) $\prod_{i=1}^{5} Q(\alpha_i) = -\frac{\text{constant term}}{\text{co-efficient of } y^5} = -23$

(ii) $\sum_{i=1}^{5} Q(\alpha_i) = -\dfrac{\text{coefficent of } y^4}{\text{coefficent of } y^5} = -10$

(iii) $\sum_{1 \le i < j \le 5}\sum Q(\alpha_i)\, Q(\alpha_j) = \dfrac{\text{coefficent of } y^3}{\text{coefficent of } y^5} = 40$

(iv) $\sum_{i=1}^{5} Q^2(\alpha_i) = \left(\sum_{i=1}^{5} Q(\alpha_i)\right)^2 - 2\sum_{1\le i<j\le 5}\sum Q(\alpha_i)Q(\alpha_j) = (-10)^2 - 2(40) = 100 - 80 = 20$

21. $(abc^2)\, x^2 + x\,(3a^2c + b^2c) - (6a^2 + ab - 2b^2) = 0$ for roots to be rational D should be perfect square

$D = (3a^2c + b^2c)^2 + 4[abc^2]\,(6a^2 + ab - 2b^2) = c^2\,[9a^4 + b^4 + 10a^2b^2 + 24a^3b - 8ab^3]$

$= c^2\,(3a^2 - b^2 + 4ab)^2 = [c(3a^2 - b^2 + 4ab)]^2$

22. $\Delta = (a^2 + b^2 + c^2)^2 - 4(a^2b^2 + b^2c^2 + c^2a^2) = a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 - 2c^2a^2$

$= a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 + 2c^2a^2 - 4c^2a^2 = (a^2 + c^2 - b^2)^2 - 4c^2a^2$

$= (a^2 + c^2 - b^2 - 2ac)\,(a^2 + c^2 - b^2 - 2ac) = [(a - c)^2 - b^2][(a + c)^2 - b^2]$

$a + c > b$ and $|a - c| < b$ (a, b, c are sides of a Δ)

$\therefore$ $(a - c)^2 < b^2$ and $(a + c)^2 > b^2$ $\Rightarrow$ $\Delta = -ve$ roots are imaginary.

23. $ax^2 + bx + c = 0$ has roots x_1

and $-ax^2 + bx + c = 0$ has root x_2

and $0 < x_1 < x_2$

Let $f(x) = ax^2 + 2bx + 2c = 0$

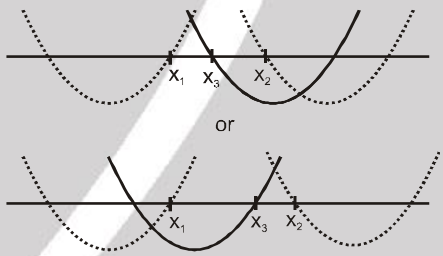
$\therefore$ $f(x_1)\, f(x_2) < 0$

L.H.S. = $(ax_1^2 + 2bx_1 + 2c)\,(ax_2^2 + 2bx_2 + 2c)$

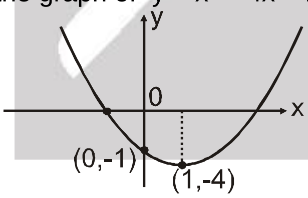
$[ax_1^2 + 2(-ax_1^2)]\,[ax_2^2 + 2(ax_2^2)]$

$(-ax_1^2)\,(3ax_2^2) = -3a^2x_1^2\,x_2^2 < 0$

$\therefore$ $f(x_1)\, f(x_2) < 0$ Hence proved.



24. Clearly the graph of $y = x^4 - 4x - 1$ is



$\therefore$ no of positive real roots = 1

Aliter

$y = x^4 - 4x - 1;\ \dfrac{dy}{dx} = 4x^3 - 4;\ \dfrac{d^2y}{dx^2} = 12x^2$ when $\dfrac{dy}{dx} = 0$, then $x = 1$

$\left.\dfrac{d^2y}{dx^2}\right|_{x=1} = 12 > 0$. so $x = 1$ is a minima point so by graph, number of positive real roots = 1

25. Since the given equation has distinct roots

$\therefore$ $D > 0$ $\Rightarrow$ $16 + 4(1 - k^2) > 0$ $\Rightarrow$ $k^2 < 5$, also $k \neq -1$

$\because$ If $k = -1$we will get only one solution, but we want two solutions

$\therefore$ $k^2 < 5$, $k \neq -1$

26. Since $\alpha, \beta = \frac{-\ b \pm \Delta}{2a}$ and $\Delta^2 = b^2 - 4ac$ either $2a\alpha + \Delta = -b$ and $2a\beta - \Delta = -b$

or $2a\alpha + \Delta = -b + 2\Delta$ and $2a\beta - \Delta = -b - 2\Delta$

$\therefore$ Sum of roots of required equation $= -2b$

And product of roots $= b^2$ or $b^2 - 4\Delta^2 = -3b^2 + 16ac$

$\therefore$ Required equation is either $x^2 + 2bx + b^2 = 0$ or $x^2 + 2bx - 3b^2 + 16ac = 0$

27. Given equation can be Expressed as

$\pi^e(x - \pi)(x - \pi - e) + e^\pi(x - e)(x - \pi - e) + (\pi^\pi + e^e)(x - e)(x - \pi) = 0$

Let $f(x) = \pi^e(x - \pi)(x - \pi - e) + e^\pi(x - e)(x - \pi - e) + (\pi^\pi + e^e)(x - e)(x - \pi)$

$\Rightarrow$ $f(e) = \pi^e(e - \pi)(-\pi) > 0$

and $f(\pi) = e^\pi(\pi - e)(-e) < 0$; hence given equation has a real root in (e, π)

again $f(\pi + e) = (\pi^\pi + e^e)\,\pi.\,e > 0$

$\because$ $\pi + e > \pi$, It concluds it has a real root in $(\pi, \pi + e)$

also $\because$ $\pi - e < e$

Hence f(x) has two real roots in $(\pi - e, \pi + e)$

28. Call roots as -2α, $\alpha + i\beta$, $\alpha - i\beta$

Sum of roots $0 = -a$ ....(1)

Sum of products taken two at a time

$-2\alpha(\alpha + i\beta) - 2\alpha(\alpha - i\beta) + \alpha^2 + \beta^2 = b$ ....(2)

Product of roots

$-2\alpha(\alpha^2 + \beta^2) = 316$ ....(3)

$\Rightarrow$ $\alpha(\alpha^2 + \beta^2) = -2 \times 79$

Case-I : $\alpha = -1$, $\alpha^2 + \beta^2 = 158$

$\Rightarrow$ $\beta^2 = 157$ not multiple of 3

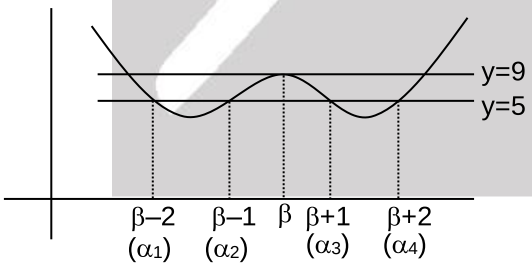
Rejected

Case-II : $\alpha = -2$, $\alpha^2 + \beta^2 = 79$ $\Rightarrow$ $\beta^2 = 75 = 3 \times 25$ $\Rightarrow$ $\beta = \pm 5\sqrt{3}$

$\therefore$ Roots are $4, -2 \pm 5\sqrt{3}$ $\Rightarrow$ $-a = 4 + (-2 + 5\sqrt{3}) + (-2 - 5\sqrt{3}) = 0$

$b = 4(-2 + 5\sqrt{3}) + (-2 + 5\sqrt{3})(-2 - 5\sqrt{3}) + 4(-2 - 5\sqrt{3}) = -16 + 4 + 75 = 63.$

29_.



$f(x) - 5 = a(x - \alpha_1)(x - \alpha_2)(x - \alpha_3)(x - \alpha_4)$

Let at $x = \beta$, $f(\beta) = 9$ then

$f(\beta) - 5 = a(\beta - \alpha_1)(\beta - \alpha_2)(\beta - \alpha_3)(\beta - \alpha_4) = 9 - 5 = 4$

$\Rightarrow$ $\beta - \alpha_1 = 2$, $\beta - \alpha_3 = -1$, $\beta - \alpha_2 = 1$, $\beta - \alpha_4 = -2$ and $a = 1$

$\Rightarrow$ $\alpha_1 = \beta - 2$, $\alpha_4 = \beta + 2$, $\alpha_2 = \beta - 1$, $\alpha_3 = \beta + 1$

$\Rightarrow$ $f(x) = (x - \beta + 2)(x - \beta - 2)(x - \beta - 1)(x - \beta + 1) + 5$

$\Rightarrow$ $f(x) = ((x - \beta)^2 - 4)((x - \beta)^2 - 1) + 5$

$\Rightarrow$ $f(x) = (x - \beta)^4 - 5(x - \beta)^2 + 9$

$\Rightarrow$ $f(x + \beta) = x^4 - 5x^2 + 9$

$\Rightarrow$ $f(\beta) = 9$ and $f'(x + \beta) = 4x^3 - 10x$

$\Rightarrow$ $f'(\beta) = 0$

30. $(xy - 7)^2 = x^2 + y^2 \Rightarrow$ $(xy - 6)^2 + 13 = (x + y)^2$ $\Rightarrow (x + y - xy + 6)(x + y + xy - 6) = 13$

Case-I $x + y + xy - 6 = 13$; $x + y - xy + 6 = 1$

On solving $(x, y) \equiv (4, 3), (3, 4)$

Case-II $x + y + xy - 6 = 1$; $x + y - xy + 6 = 13$

On solving $(x,) \neq (0, 7), (7, 0)$

In all other cases negative solutions are obtained

hence solution set is (3, 4) (4, 3), (7, 0), (0, 7)

$\therefore$ Sum of all possible values of x is $3 + 4 + 7 + 0 = 14$. **Ans.**